Area Under Curves

Question1

The area of the region bounded by the parabola $y^2 = 27x$ and the line $x = 1$ is ______ sq.units. MHT CET 2025 (5 May Shift 2)

Options:

A. $2\sqrt{3}$

B. $3\sqrt{3}$

C. $4\sqrt{3}$

D. $7\sqrt{3}$

Answer: C

Solution:

$y^2 = 27x \Rightarrow x = \frac{y^2}{27}$.

The line $x = 1$ meets the parabola at $y = \pm\sqrt{27} = \pm 3\sqrt{3}$.

Area enclosed between the line and the parabola (taking horizontal slices):

$$A = \int_{-3\sqrt{3}}^{3\sqrt{3}} \left(1 - \frac{y^2}{27}\right) dy = \left[y - \frac{y^3}{81}\right]_{-3\sqrt{3}}^{3\sqrt{3}} = 6\sqrt{3} - 2\sqrt{3} = 4\sqrt{3}.$$

So the area is $\boxed{4\sqrt{3}}$ square units.

Question2

The area of the region bounded by the curves y $= |x - 4|$, $x = 3$ and $x = 5$, and the X -axis is MHT CET 2025 (27 Apr Shift 2)

Options:

A. $\frac{21}{17}$

B. $\frac{22}{17}$

C. $\frac{23}{17}$

D. $\frac{24}{17}$

Answer: A

Question3

If the area bounded by the curve $x^2 = 4$y, X-axis and the line $x = 4$ is divided into equal areas by the line $x = \alpha$, then the value of α is ... MHT CET 2025 (26 Apr Shift 2)

Options:

A. $\frac{1}{32}$

B. 32

C. $(32)^{\frac{1}{2}}$

D. $(32)^{\frac{1}{3}}$

Answer: D

Solution:

$$x^2 = 4y \;\Rightarrow\; y = \frac{x^2}{4}$$

Total area between $x = 0$ and $x = 4$ (above the x-axis):

$$A = \int_0^4 \frac{x^2}{4}\,dx = \frac{1}{4}\cdot\frac{x^3}{3}\Big|_0^4 = \frac{64}{12} = \frac{16}{3}.$$

We need $x = \alpha$ so that the area from 0 to α is half of this:

$$\int_0^\alpha \frac{x^2}{4}\,dx = \frac{1}{2}\cdot\frac{16}{3} = \frac{8}{3}.$$

$$\frac{1}{12}\alpha^3 = \frac{8}{3} \quad\Rightarrow\quad \alpha^3 = 32 \quad\Rightarrow\quad \alpha = 32^{1/3}.$$

$$\boxed{\alpha = \sqrt[3]{32}}$$

Question4

The area of smaller part between the circle $x^2 + y^2 = 4$ and the line $x = 1$ is ________ sq.units. MHT CET 2025 (26 Apr Shift 1)

Options:

A. $\frac{4\pi}{3} - \sqrt{3}$

B. $\frac{8\pi}{3} - \sqrt{3}$

C. $\frac{4\pi}{3} + \sqrt{3}$

D. $\frac{5\pi}{3} + \sqrt{3}$

Answer: A

Solution:

$$x^2 + y^2 = 4 \Rightarrow r = 2, \quad x = 1$$

The line $x = 1$ is a chord at distance $d = 1$ from the center.
Let $\theta = \cos^{-1}(d/r) = \cos^{-1}(1/2) = \pi/3$.

The smaller region is a circular segment, whose area is

$$\text{segment} = \text{sector}(2\theta) - \text{triangle} = \frac{1}{2}r^2(2\theta) - \frac{1}{2}r^2 \sin(2\theta) = r^2\left(\theta - \frac{1}{2}\sin 2\theta\right).$$

With $r = 2,\ \theta = \pi/3,\ \sin(2\theta) = \sin(2\pi/3) = \frac{\sqrt{3}}{2}$:

$$\text{area} = 4\left(\frac{\pi}{3} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\right) = \frac{4\pi}{3} - \sqrt{3}.$$

$$\boxed{\frac{4\pi}{3} - \sqrt{3}}$$

Question5

The area bounded by the curve y $= 4x - x^2$ and X - axis in square units, is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $\frac{32}{3}$

B. 16

C. 32

D. $21\frac{1}{3}$

Answer: A

Solution:

Roots where $y = 0$: $4x - x^2 = 0 \Rightarrow x = 0, 4$.

Area between curve and x-axis:

$$A = \int_0^4 (4x - x^2)\,dx = \left[2x^2 - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}.$$

$$\boxed{\frac{32}{3}}$$

Question6

The area bounded by the parabolas y $= 9x^2$, y $= \frac{x^2}{16}$ and the line y $= 1$ is MHT CET 2025 (25 Apr Shift 1)

Options:

A. $\frac{22}{9}$ sq. units

B. $\frac{44}{9}$ sq. units

C. $\frac{8}{9}$ sq. units

D. $\frac{26}{9}$ sq. units

Answer: B

Solution:

$$\text{Curves: } y = 9x^2, \quad y = \frac{x^2}{16}, \quad y = 1.$$

For a horizontal strip at height $y \in [0, 1]$,

- right intersection with $y = \frac{x^2}{16}$: $x = 4\sqrt{y}$.
- right intersection with $y = 9x^2$: $x = \frac{1}{3}\sqrt{y}$.

Width between the parabolas on one side is $4\sqrt{y} - \frac{1}{3}\sqrt{y} = \frac{11}{3}\sqrt{y}$.
Account for both sides (left and right): width $= \frac{22}{3}\sqrt{y}$.

Area:

$$A = \int_0^1 \frac{22}{3}\sqrt{y}\, dy = \frac{22}{3} \cdot \frac{2}{3} y^{3/2}\Big|_0^1 = \frac{44}{9}.$$

$$\boxed{\frac{44}{9} \text{ sq. units}}$$

Question7

The area bounded by the curve y $= x^2 + 3$, y $= x, x = 3$ and y -axis is... MHT CET 2025 (23 Apr Shift 2)

Options:

A. $\frac{9}{2}$ sq. units

B. 18 sq. units

C. $\frac{27}{2}$ sq. units

D. $\frac{27}{3}$ sq. units

Answer: C

Solution:

Since the region is between $x = 0$ (y-axis) and $x = 3$, with top curve $y = x^2 + 3$ and bottom curve $y = x$ (because $x^2 + 3 > x$ on $[0, 3]$), the area is

$$A = \int_0^3 \left[(x^2 + 3) - x\right] dx = \left[\frac{x^3}{3} + 3x - \frac{x^2}{2}\right]_0^3 = \left(9 + 9 - \frac{9}{2}\right) - 0 = \frac{27}{2}.$$

$$\boxed{\frac{27}{2} \text{ sq. units}}$$

Question8

The area bounded by the curve $x^2 = 8\text{y}$ and the straight line $x - 8y + 2 = 0$ is MHT CET 2025 (23 Apr Shift 1)

Options:

A. $\frac{9}{8}$ sq. units

B. $\frac{15}{16}$ sq. units

C. $\frac{9}{16}$ sq. units

D. $\frac{15}{8}$ sq. units

Answer: C

Solution:

$$\text{Parabola: } x^2 = 8y \Rightarrow y = \frac{x^2}{8}, \qquad \text{Line: } x - 8y + 2 = 0 \Rightarrow y = \frac{x+2}{8}.$$

Intersections from $\frac{x^2}{8} = \frac{x+2}{8} \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, -1.$
(Then $y = \frac{4}{8} = \frac{1}{2}$ and $y = \frac{1}{8}$.)

Between $x = -1$ and $x = 2$ the line lies above the parabola, so the area is

$$A = \int_{-1}^{2} \left(\frac{x+2}{8} - \frac{x^2}{8} \right) dx = \frac{1}{8} \int_{-1}^{2} (-x^2 + x + 2)\, dx.$$

Compute:

$$\int (-x^2 + x + 2)\, dx = -\frac{x^3}{3} + \frac{x^2}{2} + 2x,$$

$$A = \frac{1}{8} \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^{2} = \frac{1}{8} \left(\frac{10}{3} - \left(-\frac{7}{6} \right) \right) = \frac{1}{8} \cdot \frac{27}{6} = \boxed{\frac{9}{16}} \text{ sq. units.}$$

Question9

If a curve $\text{y} = \text{a}\sqrt{x} + \text{b}x$ passes through the point $(1, 2)$ and the area bounded by this curve, line $x = 4$ and the X -axis is 8 sq.units, then the value of $a - b$ is MHT CET 2025 (22 Apr Shift 2)

Options:

A. -2

B. 2

C. -4

D. 4

Answer: D

Solution:

Given $y = a\sqrt{x} + bx$ passes through $(1, 2)$:

$$a + b = 2 \quad (1)$$

Area with the x-axis and $x = 4$ is the area under the curve from $x = 0$ to $x = 4$ (the curve passes through $(0, 0)$):

$$\int_0^4 (a\sqrt{x} + bx)\,dx = a \cdot \frac{2}{3}x^{3/2}\Big|_0^4 + b \cdot \frac{x^2}{2}\Big|_0^4 = \frac{16}{3}a + 8b = 8. \quad (2)$$

From (1): $b = 2 - a$. Substitute in (2):

$$\frac{16}{3}a + 8(2 - a) = 8 \Rightarrow \left(\frac{16}{3} - 8\right)a = -8 \Rightarrow -\frac{8}{3}a = -8 \Rightarrow a = 3,\ b = -1.$$

Thus,

$$a - b = 3 - (-1) = \boxed{4}.$$

Question10

The area bounded by the curve $x = 2 - y - y^2$ and the Y-axis is MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\frac{7}{6}$ sq. units

B. $\frac{13}{2}$ sq. units

C. $\frac{9}{2}$ sq. units

D. $\frac{27}{2}$ sq. units

Answer: C

Solution:

Let the region be bounded on the left by the y-axis $x = 0$ and on the right by the curve

$$x = 2 - y - y^2.$$

Where the curve meets the y-axis: $0 = 2 - y - y^2 \Rightarrow y^2 + y - 2 = 0 \Rightarrow y = -2,\ 1.$

Area (integrate w.r.t. y, from $y = -2$ to $y = 1$):

$$A = \int_{-2}^{1} x\,dy = \int_{-2}^{1} (2 - y - y^2)\,dy = \left(2y - \tfrac{y^2}{2} - \tfrac{y^3}{3}\right)_{-2}^{1} = \left(2 - \tfrac{1}{2} - \tfrac{1}{3}\right) - \left(-4 - 2 + \tfrac{8}{3}\right) = \frac{7}{6} - \left(-\frac{10}{3}\right) = \frac{27}{6} = \boxed{\frac{9}{2}} \text{sq. units.}$$

Question11

The area of the region bounded by the curve y $= |x-2|$ between $x=1, x=3$ and X -axis is ...... MHT CET 2025 (21 Apr Shift 2)

Options:

A. 1 sq.units

B. 2 sq.units

C. 3 sq.units

D. 4 sq.units

Answer: A

Solution:

$y=|x-2|$.

On $[1,2]$: $y=2-x$.
On $[2,3]$: $y=x-2$.

Area:

$$A=\int_1^2(2-x)\,dx+\int_2^3(x-2)\,dx=\left[2x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}-2x\right]_2^3=\frac{1}{2}+\frac{1}{2}=1.$$

(Geometrically: two right triangles of base 1 and height 1.)

$$\boxed{1 \text{ sq. units}}$$

Question12

The area inside the parabola $y^2=4ax$, between the lines $x=a$ and $x=4a$ is equal to MHT CET 2025 (21 Apr Shift 1)

Options:

A. $4a^2$ sq. units

B. $8a^2$ sq. units

C. $\frac{56a^2}{3}$ sq. units

D. $\frac{35a^2}{3}$ sq. units

Answer: C

Solution:

$$y^2 = 4ax \Rightarrow y = \pm 2\sqrt{ax}$$

For a vertical strip at x, the height inside the parabola is

$$(\text{top} - \text{bottom}) = 2\sqrt{ax} - (-2\sqrt{ax}) = 4\sqrt{ax}.$$

Area between $x = a$ and $x = 4a$:

$$A = \int_a^{4a} 4\sqrt{ax}\,dx = 4\sqrt{a}\int_a^{4a} x^{1/2}\,dx = 4\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_a^{4a}.$$

$$A = \frac{8}{3}\sqrt{a}\left((4a)^{3/2} - a^{3/2}\right) = \frac{8}{3}\sqrt{a}\left(8a^{3/2} - a^{3/2}\right) = \frac{8}{3}\cdot 7a^2 = \boxed{\frac{56a^2}{3}}.$$

Question13

AOB is the positive quadrant of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in which $\mathrm{OA} = 5, \mathrm{OB} = 3$. The area between the $\operatorname{arc}\mathrm{AB}$ and the chord AB of the ellipse in sq.units is MHT CET 2025 (20 Apr Shift 2)

Options:

A. $\frac{3}{5}(\pi - 2)$

B. $\frac{15}{2}(\pi - 2)$

C. $\frac{3}{10}(\pi - 2)$

D. $\frac{15}{4}(\pi - 2)$

Answer: D

Solution:

Let the ellipse be $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1.$

Intercepts: $A(5,0)$, $B(0,3)$. The chord AB has equation $y = -\frac{3}{5}x + 3$.

Area between arc AB and chord AB:

$$\int_0^5 \left(3\sqrt{1 - \frac{x^2}{25}} - \left(-\frac{3}{5}x + 3\right)\right)dx = \int_0^5 \left(3\sqrt{1 - \frac{x^2}{25}} + \frac{3}{5}x - 3\right)dx.$$

Let $x = 5\sin\theta$ so $dx = 5\cos\theta\,d\theta$, $\theta : 0 \to \frac{\pi}{2}$:

$$\int_0^5 3\sqrt{1 - \frac{x^2}{25}}\,dx = 15\int_0^{\pi/2} \cos^2\theta\,d\theta = \frac{15\pi}{4}.$$

$$\int_0^5 \left(\frac{3}{5}x - 3\right)dx = \left[\frac{3}{10}x^2 - 3x\right]_0^5 = -\frac{15}{2}.$$

Hence,

$$\text{Area} = \frac{15\pi}{4} - \frac{15}{2} = \boxed{\frac{15}{4}(\pi - 2)}.$$

Question14

The area of the region bounded by $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. $\frac{1}{2}(\pi-2)$ sq. units

B. $\frac{3}{2}(\pi-2)$ sq. units

C. $\frac{5}{4}(\pi-2)$ sq. units

D. $\frac{2}{3}(\pi-2)$ sq. units

Answer: B

Solution:

Let the ellipse be

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \quad (a=3,\ b=2),$$

and the line

$$\frac{x}{3}+\frac{y}{2}=1 \ \Rightarrow\ y=-\frac{b}{a}x+b=-\frac{2}{3}x+2.$$

This chord connects the ellipse intercepts $A(3,0)$ and $B(0,2)$.

Area between the arc AB and the chord AB (in the first quadrant) is

$$\int_0^a \left(b\sqrt{1-\frac{x^2}{a^2}}-\left(-\frac{b}{a}x+b\right)\right)dx.$$

Compute the two parts:

$$\int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx=ab\int_0^{\pi/2}\cos^2\theta\,d\theta=\frac{\pi ab}{4},$$

$$\int_0^a\left(-\frac{b}{a}x+b\right)dx=\left[-\frac{b}{2a}x^2+bx\right]_0^a=\frac{ab}{2}.$$

Hence the desired area is

$$\frac{\pi ab}{4}-\frac{ab}{2}=\frac{ab}{4}(\pi-2)=\frac{3\cdot 2}{4}(\pi-2)=\boxed{\frac{3}{2}(\pi-2)}.$$

Question15

The area enclosed between the curves $y^2=4x$ and $y=|x|$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. $\frac{8}{3}$ sq. units

B. $\frac{5}{3}$ sq. units

C. $\frac{4}{3}$ sq. units

D. $\frac{2}{3}$ sq. units

Answer: A

Solution:

$y^2 = 4x \Rightarrow x = \frac{y^2}{4}$.

The V-curve $y = |x|$ in the upper half-plane is the line $y = x$ (since $y \geq 0$).

They intersect at $(0,0)$ and $(4,4)$.

Use horizontal slices (integrate in y) from $y = 0$ to $y = 4$.

Right boundary: $x = \frac{y^2}{4}$; left boundary: $x = y$.

Width $= \frac{y^2}{4} - y$ (take absolute value inside the integral).

$$A = \int_0^4 \left(y - \frac{y^2}{4}\right) dy = \left[\frac{y^2}{2} - \frac{y^3}{12}\right]_0^4 = 8 - \frac{64}{12} = \frac{8}{3}.$$

$$\boxed{\frac{8}{3} \text{ sq. units}}$$

Question16

The ratio of the areas bounded by the curves y $= \cos x$ and y $= \cos 2x$ between $x = 0, x = \frac{\pi}{3}$ and X -axis is MHT CET 2025 (19 Apr Shift 1)

Options:

A. $\sqrt{2} : 1$

B. $1 : 1$

C. $2 : 1$

D. $1 : 3$

Answer: C

Solution:

$$A_1 = \int_0^{\pi/3} \cos x \, dx = \sin x \Big|_0^{\pi/3} = \frac{\sqrt{3}}{2}$$

$$A_2 = \int_0^{\pi/3} \cos 2x \, dx = \frac{1}{2} \sin 2x \Big|_0^{\pi/3} = \frac{1}{2} \cdot \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{4}$$

$$\frac{A_1}{A_2} = \frac{\sqrt{3}/2}{\sqrt{3}/4} = 2 : 1.$$

Answer: $2 : 1$.

Question17

The area (in sq. units) bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the line $y = 2$ is MHT CET 2024 (16 May Shift 2)

Options:

A. $20\sqrt{2}$

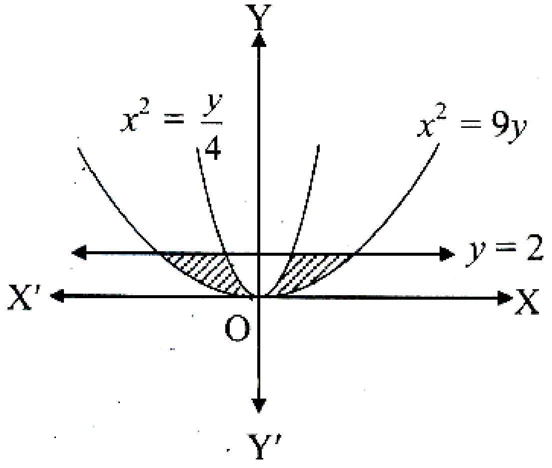
B. $\frac{10\sqrt{2}}{3}$

C. $\frac{20\sqrt{2}}{3}$

D. $10\sqrt{2}$

Answer: C

Solution:



Required area

$$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right) dy$$

$$= 2\left[3\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 - \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2\right]$$

$$= 2\left[2\left(2^{\frac{3}{2}} - 0\right) - \frac{1}{3}\left(2^{\frac{3}{2}} - 0\right)\right]$$

$$= 2\left[2(2\sqrt{2}) - \frac{1}{3}(2\sqrt{2})\right]$$

$$= \frac{20\sqrt{2}}{3}$$

Question18

The area (in sq. units) of the region described by $\{(x, y) / y^2 \le 2x \text{ and } y \ge (4x - 1)\}$ is MHT CET 2024 (16 May Shift 1)

Options:

A. $\frac{15}{64}$

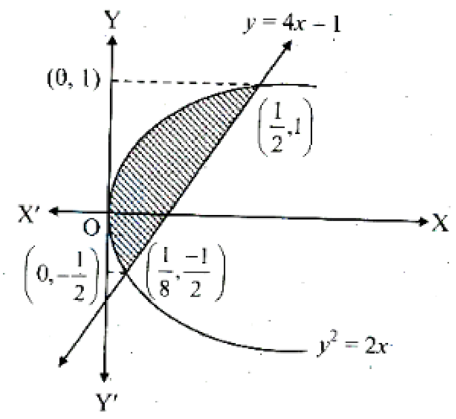
B. $\frac{9}{32}$

C. $\frac{7}{32}$

D. $\frac{5}{64}$

Answer: B

Solution:



Putting $x = \frac{y^2}{2}$ in $y = 4x - 1$, we get

$$y = 4\left(\frac{y^2}{2}\right) - 1 \Rightarrow 2y^2 - y - 1 = 0$$
$$\Rightarrow (y-1)(2y+1) = 0$$
$$\Rightarrow y = 1, \frac{-1}{2}$$

$\therefore$ Required area

$$= \int_{-1/2}^{1}\left(\frac{y+1}{4}\right) dy - \int_{-1/2}^{1} \frac{y^2}{2} dy$$
$$= \frac{1}{4}\left[\frac{y^2}{2} + y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$
$$= \frac{1}{4}\left[\left(\frac{1}{2} - \frac{1}{8}\right) + \left(1 + \frac{1}{2}\right)\right] - \frac{1}{2}\left(\frac{1}{3} + \frac{1}{24}\right)$$
$$= \frac{1}{4}\left(\frac{15}{8}\right) - \frac{1}{2}\left(\frac{9}{24}\right)$$
$$= \frac{15}{32} - \frac{3}{16}$$
$$= \frac{9}{32}$$

Question19

The area enclosed between the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is MHT CET 2024 (15 May Shift 2)

Options:

A. $\frac{17}{3}$ sq. units

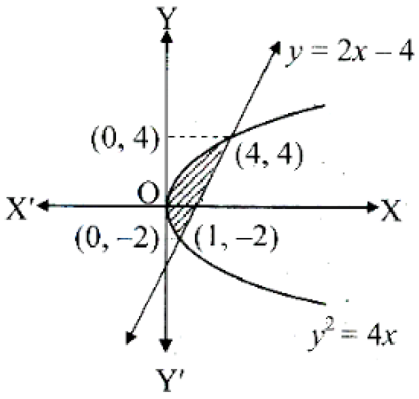
B. 15 sq. units

C. $\frac{19}{3}$ sq. units

D. 9 sq. units

Answer: D

Solution:



Putting $x = \frac{y^2}{4}$ in $y = 2x - 4$, we get

$$y = 2\left(\frac{y^2}{4}\right) - 4$$
$$\Rightarrow y^2 - 2y - 8 = 0$$
$$\Rightarrow (y-4)(y+2) = 0$$
$$\Rightarrow y = 4, -2$$

$$\therefore \quad \text{Required area } = \int_{-2}^{4}\left(\frac{y+4}{2} - \frac{y^2}{4}\right) dy$$
$$= \frac{1}{2}\left[\frac{y^2}{2} + 4y\right]_{-2}^{4} - \frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^{4}$$
$$= \frac{1}{2}[8 + 16 - (2-8)] - \frac{1}{12}[64 - (-8)]$$
$$= 15 - 6$$
$$= 9 \text{ sq. units}$$

Question20

The area of the region bounded by curves $y = 3x + 1, y = 4x + 1$ and $x = 2$ is MHT CET 2024 (15 May Shift 1)

Options:

A. 1 sq. units

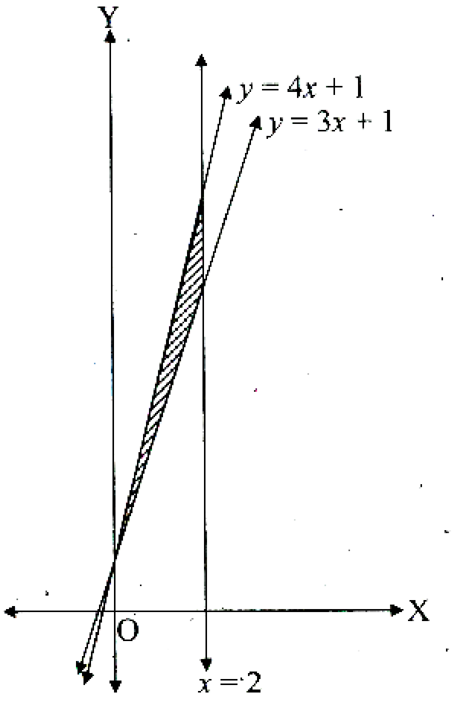
B. 2 sq. units

C. 3 sq. units

D. 4 sq. units

Answer: B

Solution:



$$\text{Required area } = \int_0^2 [4x + 1 - (3x + 1)] \mathrm{d}x$$

$$= \int_0^2 x \, \mathrm{d}x = \left[\frac{x^2}{2}\right]_0^2 = 2 \text{ sq. units}$$

Question21

If two sides of a square are $4x + 3y - 20 = 0$ and $4x + 3y + 15 = 0$, then the area of the square is MHT CET 2024 (15 May Shift 1)

Options:

A. 36 sq. units

B. 16 sq. units

C. 4 sq. units

D. 49 sq. units

Answer: D

Solution:

Given equations of lines are $4x+3y-20=0$ and $4x+3y+15=0$

Slope of $4x+3y-20=0$ is $\frac{-4}{3}$.

Slope of $4x+3y+15=0$ is $\frac{-4}{3}$

$\therefore$ Lines are parallel.

$\therefore$ Distance between two parallel lines

$$=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$
$$=\left|\frac{-20-15}{\sqrt{4^2+3^2}}\right|$$
$$=\frac{35}{5}=7 \text{ units}$$

$\therefore$ Area of square $=7^2=49$ sq. units

Question22

The area (in sq. units) of the region bounded by $y-x=2$ and $x^2=y$ is equal to MHT CET 2024 (11 May Shift 2)

Options:

A. $\frac{2}{3}$

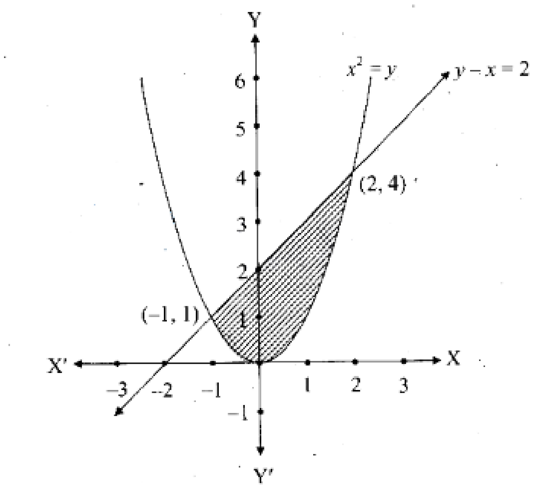
B. $\frac{4}{3}$

C. $\frac{9}{2}$

D. $\frac{16}{3}$

Answer: C

Solution:



$$y-x=2 \text{ and } x^2=y$$
$$\therefore \quad x^2-x-2=0$$
$$\therefore \quad (x-2)(x+1)=0$$
$$\therefore \quad x=2 \text{ or } x=-1$$
$$\therefore \quad y=4, y=1$$

$\therefore$ Points of intersection of the two curves are

- $(2,4)$ and $(-1,1)$

$$\begin{aligned} \text{Required area } &= \int_{-1}^{2}(2+x)-x^2 dx \\ &= 2[x]_{-1}^{2}+\frac{1}{2}\left[x^2\right]_{-1}^{2}-\frac{1}{3}\left[x^3\right]_{-1}^{2} \\ &= 6+\frac{3}{2}-3=\frac{9}{2} \text{ sq. units} \end{aligned}$$

Question23

The area of the region lying in the first quadrant by $y=4x^2, x=0, y=2, y=4$ is MHT CET 2024 (11 May Shift 1)

Options:

A. $\frac{1}{6}[8-2\sqrt{2}]$ sq.units

B. $\frac{1}{3}[8-2\sqrt{2}]$ sq.units

C. $[8-2\sqrt{2}]$ sq.units

D. $[8+2\sqrt{2}]$ sq.units

Answer: B

Solution:

$$\begin{aligned} \text{Required area } &= \int_{2}^{4}\frac{\sqrt{y}}{2}\, \mathrm{d}y \\ &= \frac{1}{2}\frac{\left[y^{\frac{3}{2}}\right]_{2}^{4}}{\frac{3}{2}} \\ &= \frac{1}{3}(8-2\sqrt{2}) \text{ sq. units} \end{aligned}$$

Question24

The area (in sq. units) of the region $\{(x, y)/x \geq 0, x+y \leq 3, x^2 \leq 4y$ and $y \leq 1+\sqrt{x}\}$ is MHT CET 2024 (10 May Shift 2)

Options:

A. $\frac{9}{2}$

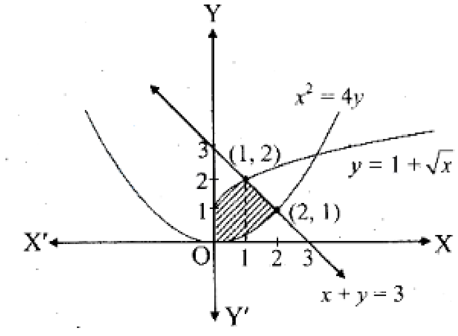
B. $\frac{3}{2}$

C. $\frac{7}{2}$

D. $\frac{5}{2}$

Answer: D

Solution:



Given inequalities are

$x \geq 0$
$x+y \leq 3$
$x^2 \leq 4y,$
$y \leq 1+\sqrt{x}$

$\therefore$ The equalities are

$x+y=3 \ldots(i)$
$x^2=4y \ldots(ii)$
$y=1+\sqrt{x} \ldots(iii)$

from (i) and (iii), we get

$3-x=1+\sqrt{x}$
$\therefore \quad x+\sqrt{x}-2=0$

$\therefore \quad (\sqrt{x}+2)(\sqrt{x}-1)=0$
$\Rightarrow \sqrt{x}=1 \quad \ldots[\because \sqrt{x}$ cannot be negative $]$
$\Rightarrow x=1$ and $y=2$

From (i) and (ii), we get

$x+\frac{x^2}{4}=3$

$\therefore \quad x^2+4x-12=0$

$\therefore \quad (x+6)(x-2)=0$

$\Rightarrow x=2$

$\Rightarrow y=1$

$\ldots[\because x \geq 0]$

$\therefore$ Required area

$$
\begin{aligned}
& =\int_0^1\left(1+\sqrt{x}-\frac{x^2}{4}\right) \mathrm{d} x+\int_1^2\left(3-x-\frac{x^2}{4}\right) \mathrm{d} x \\
& =\int_0^1(1+\sqrt{x}) \mathrm{d} x+\int_1^2(3-x)-\frac{1}{4} \int_0^2 x^2 \mathrm{~d} x \\
& =[x]_0^1+\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1+3[x]_1^2-\frac{1}{2}\left[x^2\right]_1^2-\frac{1}{12}\left[x^3\right]_0^2 \\
& =1+\frac{2}{3}+3-\frac{3}{2}-\frac{2}{3} \\
& =\frac{5}{2}
\end{aligned}
$$

Question25

Area (in sq.units) lying in the first quadrant and bounded by the circle $x^2+y^2=4$ and the lines $x=0$ and $x=2$ is MHT CET 2024 (10 May Shift 1)

Options:

A. π

B. $\frac{\pi}{2}$

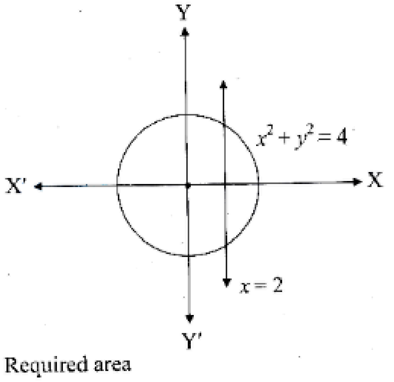
C. $\frac{\pi}{3}$.

D. $\frac{\pi}{4}$

Answer: A

Solution:

$x^2 + y^2 = 4$
$\Rightarrow y^2 = 4 - x^2$



Required area

$= \int_0^2 \sqrt{4 - x^2} dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\cdot\frac{x}{2}\right)\right]_0^2$

$= \left[\frac{2}{2}\sqrt{4 - (2)^2} + \frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right) - \frac{0}{2}\sqrt{4 - 0} - \frac{4}{2}\sin^{-1}\left(\frac{0}{2}\right)\right]$

$= 2 \times \frac{\pi}{2}$

$= \pi$ sq.

Question26

The area (in sq. units), in the first quadrant bounded by the curve $y = x^2 + 2$ and the lines $y = x + 1, x = 0$ and $x = 2$, is MHT CET 2024 (09 May Shift 2)

Options:

A. $\frac{1}{3}$

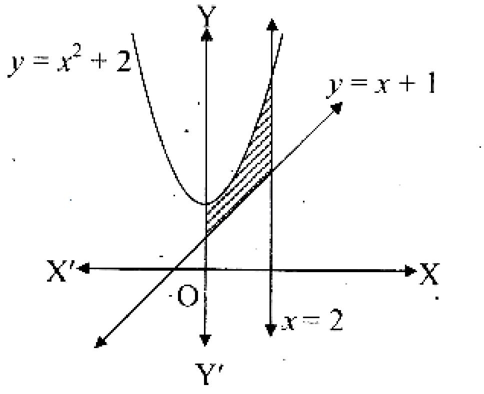
B. $\frac{2}{3}$

C. $\frac{5}{3}$

D. $\frac{8}{3}$

Answer: D

Solution:



$$\text{Required area } = \int_0^2 \left[\left(x^2+2\right)-(x+1)\right] \mathrm{d}x$$

$$= \int_0^2 \left(x^2-x+1\right) \mathrm{d}x$$

$$= \left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^2$$

$$= \frac{8}{3}-2+2-0$$

$$= \frac{8}{3} \text{ sq. units}$$

Question27

The area (in sq. units) bounded by the curves $y=(x+1)^2, y=(x-1)^2$ and the line $y=\frac{1}{4}$ is MHT CET 2024 (09 May Shift 1)

Options:

A. $\frac{2}{3}$

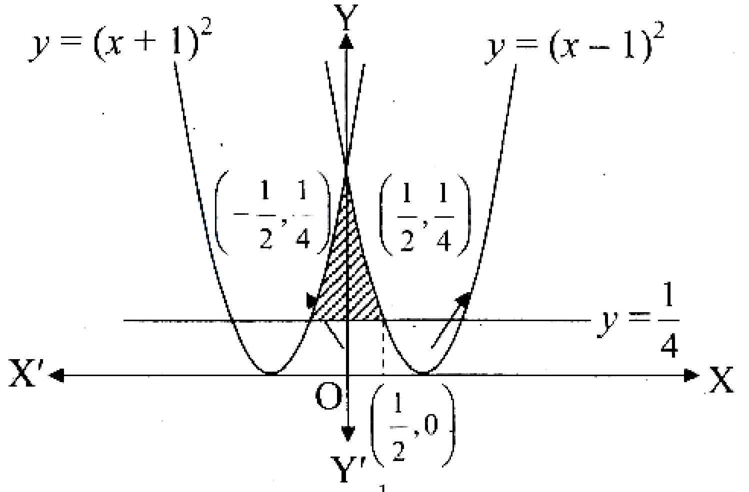
B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: C

Solution:



$$\text{Required area } = 2\int_0^{\frac{1}{2}}\left[(x-1)^2-\frac{1}{4}\right]\mathrm{d}x$$

$$= 2\left[\frac{(x-1)^3}{3}\right]_0^{\frac{1}{2}} - \frac{1}{2}[x]_0^{\frac{1}{2}}$$

$$= \frac{2}{3}\left(-\frac{1}{8}+1\right) - \frac{1}{2}\left(\frac{1}{2}-0\right)$$

$$= \frac{1}{3} \text{ sq. units}$$

Question28

The area (in square units) in the first quadrant bounded by the curve $y=x^2+2$ and the lines $y=x+1, x=0$ and $x=3$, is MHT CET 2024 (04 May Shift 2)

Options:

A. $\frac{15}{4}$

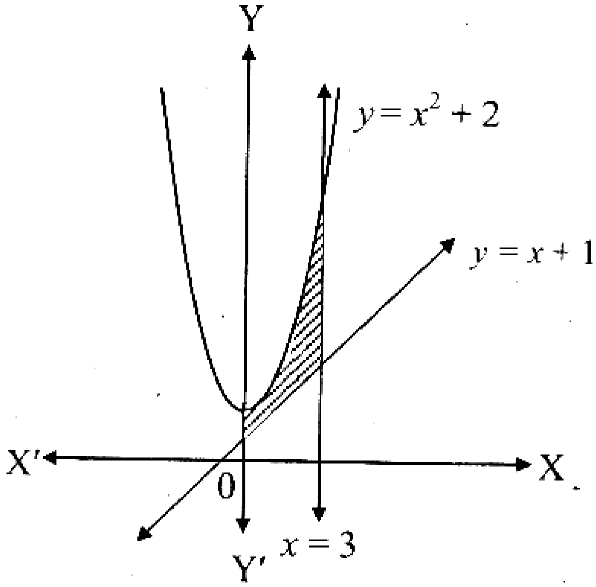
B. $\frac{21}{2}$

C. $\frac{17}{4}$

D. $\frac{15}{2}$

Answer: D

Solution:



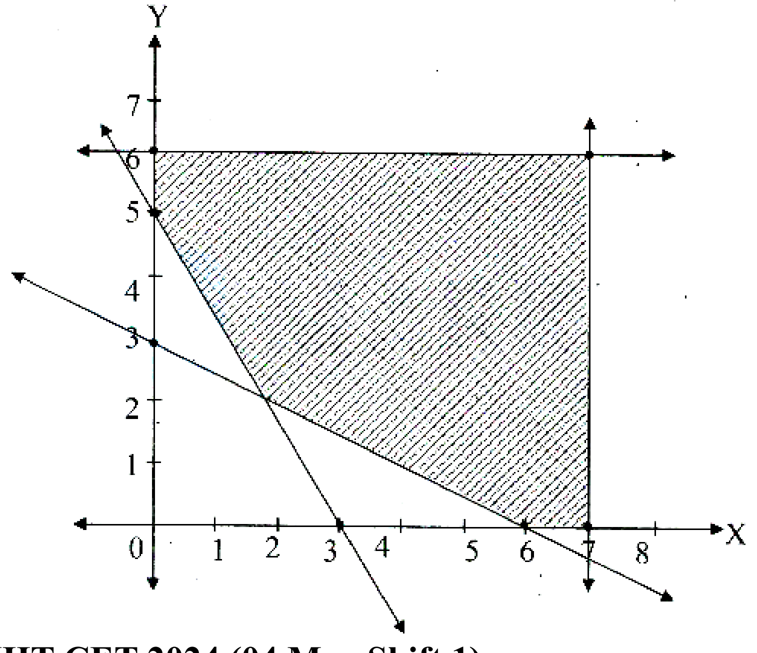
$$\text{Required area } = \int_0^3 \left(x^2 + 2\right) - (x + 1) \mathrm{d}x$$

$$= \int_0^3 \left(x^2 - x + 1\right) \mathrm{d}x$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 = \frac{15}{2} \text{ sq. units}$$

Question29

The shaded region in the following figure is the solution set of the inequations



MHT CET 2024 (04 May Shift 1)

Options:

A. $x + 2y \leq 6, 5x + 3y \geq 15, x \leq 7, y \leq 6, x,$
$y \geq 0$

B. $x + 2y \geq 6, 5x + 3y \geq 15, x \leq 7, y \leq 6, x$
$y \geq 0$

C. $x + 2y \geq 6, 5x + 3y \leq 15, x \geq 7, y \leq 6, x,$
$y \geq 0$

D. $x + 2y \leq 6, 5x + 3y \leq 15, x \leq 7, y \geq 6, x,$
$y \geq 0$

Answer: A

Solution:

To determine which option corresponds to the shaded region in the figure, let's analyze each inequality represented in the answer choices.

Axes and the Region: The region is bounded by the axes $x \geq 0$ and $y \geq 0$, which are present in all options. The other inequalities will help define the upper bounds more clearly.

Inequalities Exploration:

Option (1):

$x + 2y \leq 6$ (below the line)

$5x + 3y \geq 15$ (above the line)

$x \leq 7$ (to the left of the vertical line)

$y \leq 6$ (below the horizontal line)

Option (2):

$x + 2y \geq 6$ (above the line)

$5x + 3y \geq 15$ (above the line)

Options continue in the same manner with x and y.

Interpreting the Shaded Region:

Examine if the inequalities allow a bounded area in the first quadrant.

The acceptable area must lie below the line for $x + 2y \leq 6$ and above the line for $5x + 3y \geq 15$.

Summary of Options:

Option (1) fulfills the conditions that keep the region bounded below $y = 6$, to the left of $x = 7$, and within the first quadrant.

Following the analysis, the correct answer corresponds to the region described in Option (1):

$$(1) \quad x + 2y \leq 6, \quad 5x + 3y \geq 15, \quad x \leq 7, \quad y \leq 6, \quad x, y \geq 0$$

Question30

The area bounded between the curves $y = ax^2$ and $x = \mathrm{a}y^2(\mathrm{a} > 0)$ is 1 sq. units, then the value of a is MHT CET 2024 (04 May Shift 1)

Options:

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{2}$

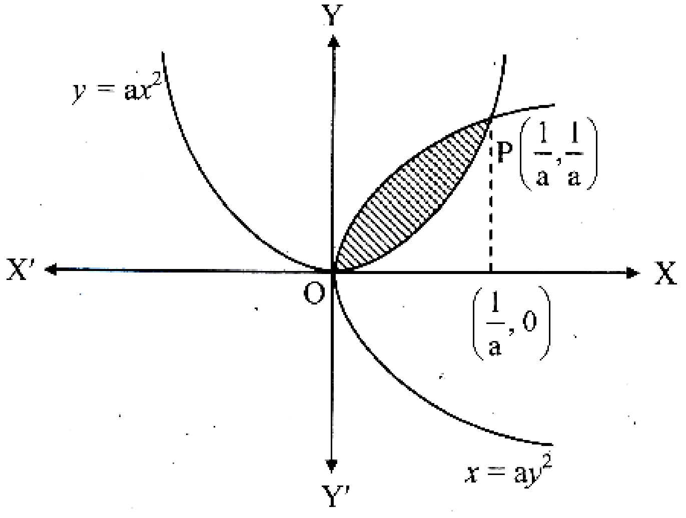
C. 1

D. $\frac{1}{3}$

Answer: A

Solution:

The two curves intersect at $O(0,0)$ and $P\left(\frac{1}{\mathrm{a}}, \frac{1}{\mathrm{a}}\right)$.



According to the given condition,

$$\int_0^{\frac{1}{2}} \left(\sqrt{\frac{x}{\mathrm{a}}} - \mathrm{a}x^2\right) \mathrm{d}x = 1$$

$$\Rightarrow \left[\frac{2}{3\sqrt{\mathrm{a}}} x^{3/2} - \frac{\mathrm{a}x^3}{3}\right]_0^{1/\mathrm{a}} = 1$$

$$\Rightarrow \frac{2}{3\sqrt{\mathrm{a}}} \times \frac{1}{\mathrm{a}^{3/2}} - \frac{\mathrm{a}}{3} \times \frac{1}{\mathrm{a}^3} = 1$$

$$\Rightarrow \frac{2}{3\mathrm{a}^2} - \frac{1}{3\mathrm{a}^2} = 1 \Rightarrow \frac{1}{3\mathrm{a}^2} = 1$$

$$\Rightarrow \mathrm{a} = \frac{1}{\sqrt{3}} \quad \ldots [\because \mathrm{a} > 0]$$

Question31

The area of the region, bounded by the parabola $y = x^2 + 2$ and the lines $y = x, x = 0$ and $x = 3$, is MHT CET 2024 (03 May Shift 2)

Options:

A. $\frac{9}{2}$ sq. units

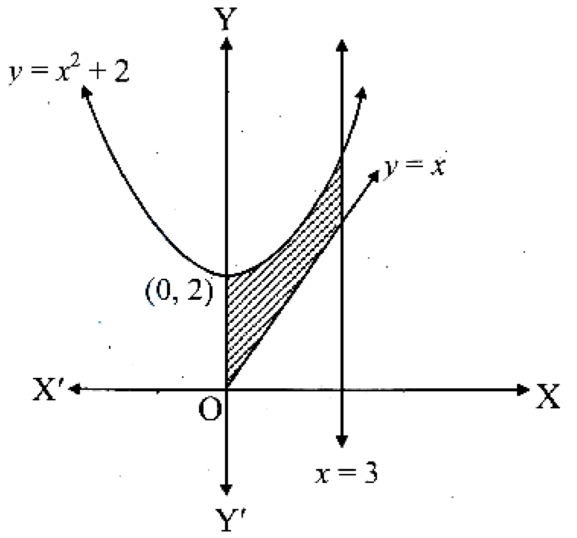
B. $\frac{11}{2}$ sq. units

C. $\frac{15}{2}$ sq. units

D. $\frac{21}{2}$ sq. units

Answer: D

Solution:



$$\text{Required area } = \int_0^3 (x^2 + 2 - \dot{x})\, dx$$

$$= \left[\frac{x^3}{3} + 2x - \frac{x^2}{2}\right]_0^3$$

$$= 9 + 6 - \frac{9}{2} - 0$$

$$= \frac{21}{2} \text{ sq. unit}$$

Question32

The maximum value of $Z = x + y$, subjected to $x + y \leq 10, 5x + 3y \geq 15, x \leq 6, x, y \geq 0$ MHT CET 2024 (03 May Shift 2)

Options:

A. occurs only at unique point

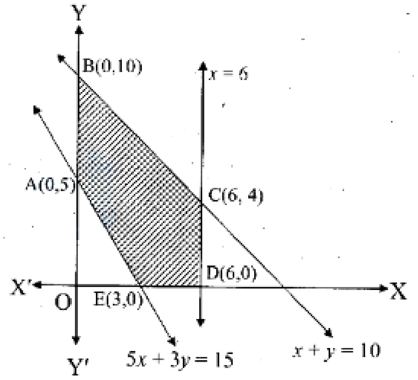
B. occurs only at two distinct points

C. occurs at infinitely many points

D. does not exist

Answer: C

Solution:



Feasible region lies on the origin side of $x+y=10, x=6$ and non-origin side of $5x+3y=15$

The corner points of feasible region are $\mathrm{A}(0,5)$ and $\mathrm{B}(0,10), \mathrm{C}(6,4), \mathrm{D}(6,0), \mathrm{E}(3,0)$

At $\mathrm{A}(0,5), \mathrm{z}=0+5=5$

At B(0,10), $\mathrm{z}=0+10=10$

At C(6,4), $z=6+4=10$

At $\mathrm{D}(6,0), \mathrm{z}=6+0=6$

At $E(3,0), z=3+0=3$

$\therefore$ z has maximum value at $\mathrm{B}(0,10)$ and $\mathrm{C}(6,4)$.

$\therefore$ z has infinite solution on seg BC .

Question33

The area (in sq. units) of the region bounded by the curve $x^2=4y$ and the straight line $x=4y-2$ is MHT CET 2024 (03 May Shift 1)

Options:

A. $\frac{9}{8}$

B. $\frac{7}{8}$

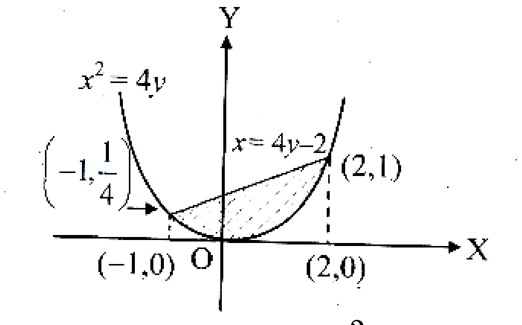
C. $\frac{5}{4}$

D. $\frac{3}{4}$

Answer: A

Solution:

The points of intersection of $x^2 = 4y$ and $x = 4y - 2$ are $(2,1)$ and $\left(-1, \frac{1}{4}\right)$.



$$= \int_{-1}^{2} \frac{1}{4}(x+2)\mathrm{d}x - \int_{-1}^{2} \frac{1}{4}x^2 \, \mathrm{d}x$$

Required area $= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$$= \frac{9}{8} \text{ sq. units}$$

Question34

The area (in sq. units) bounded by the curves $y = \sqrt{x}, 2y - x + 3 = 0$, X-axis and lying in the first quadrant is MHT CET 2024 (02 May Shift 2)

Options:

A. 36

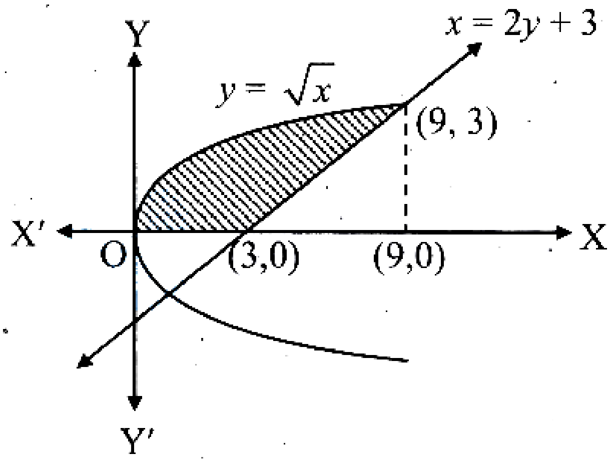
B. 18

C. $\frac{27}{4}$

D. 9

Answer: D

Solution:



$$\text{Required area } = \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx$$
$$= \left[\frac{2x^{3/2}}{3}\right]_0^9 - \frac{1}{2}\left[\frac{x^2}{2} - 3x\right]_3^9$$
$$= \frac{2}{3}(27-0) - \frac{1}{2}(36-18)$$
$$= 9 \text{ sq.units}$$

Question35

The area bounded by the curve $y = |x-2|, x = 1, x = 3$ and X-axis is MHT CET 2023 (14 May Shift 2)

Options:

A. 3 sq. units

B. 2 sq. units

C. 1 sq. units

D. 4 sq. units

Answer: C

Solution:

$$\text{Required area } = \int_1^3 |x-2| dx$$

$$= \int_1^2 (2-x)\mathrm{d}x + \int_2^3 (x-2)\mathrm{d}x$$
$$= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$
$$= \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. unit}$$

Question36

The area bounded by the curves $y=(x-1)^2$, $y=(x+1)^2$ and $y=\frac{1}{4}$ is MHT CET 2023 (14 May Shift 1)

Options:

A. $\frac{1}{3}$ sq. units.

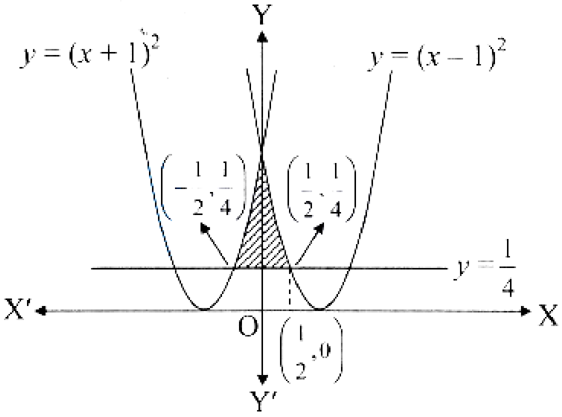
B. $\frac{2}{3}$ sq. units.

C. $\frac{1}{4}$ sq. units.

D. $\frac{1}{5}$ sq. units.

Answer: A

Solution:



$$\text{Required area } = 2\int_0^{\frac{1}{2}} \left[(x-1)^2 - \frac{1}{4}\right] \mathrm{d}x$$

$$= 2\left[\frac{(x-1)^3}{3}\right]_0^{\frac{1}{2}} - \frac{1}{2}[x]_0^{\frac{1}{2}}$$
$$= \frac{2}{3}\left(-\frac{1}{8}+1\right) - \frac{1}{2}\left(\frac{1}{2}-0\right)$$
$$= \frac{1}{3} \text{ sq. units}$$

Question37

If a curve $y = \mathrm{a}\sqrt{x} + \mathrm{b}x$ passes through the point $(1, 2)$ and the area bounded by the curve, line $x = 4$ and X-axis is 8 sq. units, then MHT CET 2023 (13 May Shift 2)

Options:

A. $\mathrm{a} = 3,\ \mathrm{b} = -1$

B. $\mathrm{a} = 3,\ \mathrm{b} = 1$

C. $\mathrm{a} = -3,\ \mathrm{b} = 1$

D. $\mathrm{a} = -3,\ \mathrm{b} = -1$

Answer: A

Solution:

The given curve passes through $(1, 2)$.

$\therefore \quad 2 = a + b$

According to the given condition,

$\int_0^4 (a\sqrt{x} + bx)dx = 8$ ...(i)

$\Rightarrow \frac{2\mathrm{a}}{3}\left[x^{3/2}\right]_0^4 + \frac{\mathrm{b}}{2}\left[x^2\right]_0^4 = 8 \Rightarrow \frac{2\mathrm{a}}{3} \cdot 8 + 8\,\mathrm{b} = 8$

$\Rightarrow 2a + 3b = 3$ ...(ii)

From (i) and (ii), we get

$\mathrm{a} = 3,\ \mathrm{b} = -1$

Question38

The area (in sq. units) of the region bounded by curves $y=3x+1, y=4x+1$ and $x=3$ is MHT CET 2023 (13 May Shift 1)

Options:

A. $\frac{7}{2}$

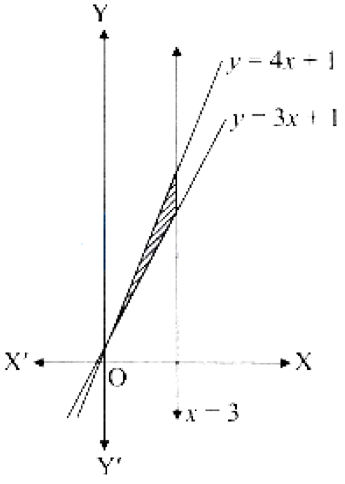
B. $\frac{9}{5}$

C. $\frac{9}{2}$

D. $\frac{7}{5}$

Answer: C

Solution:



$$\text{Required area } = \cdot \int_0^3 [4x+1-(3x+1)]\mathrm{d}x$$

$$= \int_0^3 x \, \mathrm{d}x$$

$$= \left[\frac{x^2}{2}\right]_0^3$$

$$= \frac{9}{2} \text{ sq. units}$$

Question39

The area (in sq. units) of the smaller part of the circle $x^2+y^2=\mathrm{a}^2$ cut off by the line $x=\frac{\mathrm{a}}{\sqrt{2}}$ is MHT CET 2023 (12 May Shift 2)

Options:

A. $\frac{\mathrm{a}^2}{4}\left|\frac{\pi}{2}-1\right|$

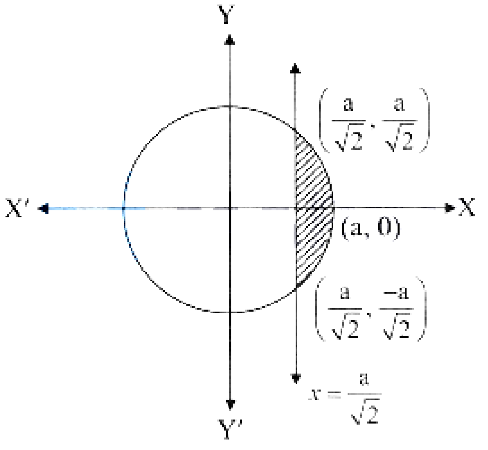
B. $a^2\left|\frac{\pi}{4}-1\right|$

C. $\frac{\mathrm{a}^2}{2}\left|\frac{\pi}{2}-1\right|$

D. $\frac{a^2}{4}\left|\frac{\pi}{4}-1\right|$

Answer: C

Solution:



Substitute $x=\frac{\mathrm{a}}{\sqrt{2}}$ in $x^2+y^2=\mathrm{a}^2$, we get $\frac{\mathrm{a}^2}{2}+y^2=\mathrm{a}^2 \Rightarrow y=\pm\frac{\mathrm{a}}{\sqrt{2}} \therefore$

Required area

$$=2\int_{\frac{a}{\sqrt{2}}}^{a}\sqrt{a^2-x^2}dx$$

$$=2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{\frac{a}{\sqrt{2}}}^{2}$$

$$=2\left\{\left[0+\frac{a^2}{2}\times\frac{\pi}{2}\right]-\left[\frac{a}{2\sqrt{2}}\sqrt{a^2-\frac{a^2}{2}}+\frac{a^2}{2}\times\frac{\pi}{4}\right]\right\}$$

$$= 2\left[\frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}\right]$$
$$= \frac{a^2}{2}\left|\pi - 1 - \frac{\pi}{2}\right|$$
$$= \frac{a^2}{2}\left|\frac{\pi}{2} - 1\right|$$

Question40

The area of the region bounded by the curves $y = \mathrm{e}^x, y = \log x$ and lines $x = 1, x = 2$ is MHT CET 2023 (12 May Shift 1)

Options:

A. $(\mathrm{e} - 1)^2$sq. units

B. $\left(\mathrm{e}^2 - \mathrm{e} + 1\right)$ sq. units

C. $\left(\mathrm{e}^2 - \mathrm{e} + 1 - 2\log 2\right)$ sq. units

D. $\left(\mathrm{e}^2 + \mathrm{e} - 2\log 2\right)$ sq. units

Answer: C

Solution:

Required Area
$$= \int_1^2 (\mathrm{e}^x - \log x)\, \mathrm{d}x$$
$$= [\mathrm{e}^x]_1^2 - \int_1^2 1 \log x \ \mathrm{d}x$$
$$= \left(\mathrm{e}^2 - \mathrm{e}\right) - \left[x \log x - \int_1^2 1 \ \mathrm{d}x\right]$$
$$= \left(\mathrm{e}^2 - \mathrm{e}\right) - [x \log x - x]_1^2$$

$$= \left(e^2 - e\right) - [(2\log 2 - 2) - (1\log 1 - 1)]$$
$$= e^2 - e - (2\log 2 - 2 - 0 + 1)$$
$$= e^2 - e - (2\log 2 - 1)$$
$$= \left(e^2 - e + 1 - 2\log 2\right) \text{ sq. units}$$

Question41

The area of the region bounded by the parabola $y = x^2$ and the curve $y = |x|$ is MHT CET 2023 (11 May Shift 1)

Options:

A. $\frac{1}{2}$ sq. units

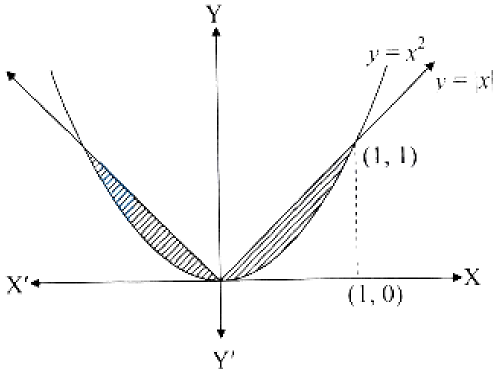
B. $\frac{1}{3}$ sq. units

C. $\frac{1}{4}$ sq. units

D. $\frac{1}{6}$ sq. units

Answer: B

Solution:



$$\text{Required area } = 2\int_0^1 \left(x - x^2\right) dx$$
$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$
$$= 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3} \text{ sq.units}$$

Question42

The area bounded by the X-axis and the curve $y = x(x-2)(x+1)$ is MHT CET 2023 (10 May Shift 2)

Options:

A. $\frac{37}{12}$ sq. units

B. $\frac{27}{12}$ sq. units

C. $\frac{37}{4}$ sq. units

D. $\frac{27}{13}$ sq. units

Answer: A

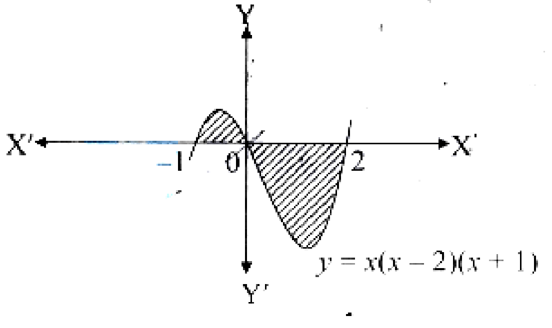
Solution:

For X-axis,

$y = 0$

$\therefore \quad x(x-2)(x+1) = 0$

$\Rightarrow x = 0$ or $x = 2$ or $x = -1$



$$\text{Required area} = \int_{-1}^{0} y\,dx + \left|\int_{0}^{2} y\,dx\right|$$

$$= \int_{-1}^{0} x(x-2)(x+1)dx + \left|\int_{0}^{2} x(x-2)(x+1)dx\right|$$

$$= \int_{-1}^{0} (x^3 - x^2 - 2x)\,dx + \left|\int_{0}^{2} (x^3 - x^2 - 2x)\,dx\right|$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^{0} + \left|\left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{0}^{2}\right|$$

$$= \frac{5}{12} + \left|-\frac{8}{3}\right|$$

$$= \frac{37}{12} \text{ sq. units}$$

Question43

Area of the region bounded by the curve $y=\sqrt{49-x^2}$ and X-axis is MHT CET 2023 (10 May Shift 1)

Options:

A. 49π sq. units

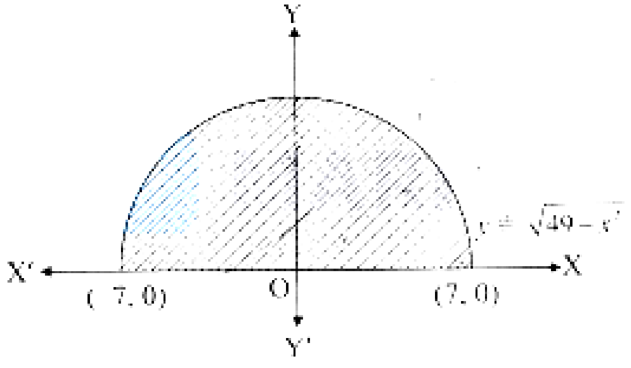
B. $\frac{49\pi}{2}$ sq. units

C. $\frac{49\pi}{4}$ sq. units

D. 98π sq. units

Answer: B

Solution:



$$
\begin{aligned}
\text{Required area } &= 2\int_0^7 \sqrt{49-x^2}\, dx \\
&= 2\left[\frac{x}{2}\sqrt{49-x^2}+\frac{49}{2}\sin^{-1}\left(\frac{x}{7}\right)\right]_0^7 \\
&= 2\left[\frac{7}{2}\sqrt{49-49}+\frac{49}{2}\sin^{-1}\left(\frac{7}{7}\right)\right]-0 \\
&= 2\times\frac{49}{2}\times\frac{\pi}{2} \\
&= \frac{49\pi}{2} \text{ sq. units}
\end{aligned}
$$

Question44

The area (in sq. units) bounded by the curve $y = x|x|$, X-axis and the lines $x = -1$ and $x = 1$ is MHT CET 2023 (09 May Shift 2)

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D. $\frac{4}{3}$

Answer: A

Solution:

$y = x|x| \ldots [Given]$

Required area

$$= \int_{-1}^{1} x|x| \mathrm{d}x$$
$$= 2\int_{0}^{1} x^2 \, \mathrm{d}x \quad \ldots [\because \text{ Area is always positive }]$$
$$= 2 \times \left[\frac{x^3}{3}\right]_0^1$$
$$= 2 \times \left(\frac{1}{3} - 0\right) = \frac{2}{3} \text{ sq.units}$$

Question45

The area (in sq. units) of the region $A = \left\{(x, y) / \frac{y^2}{2} \leq x \leq y + 4\right\}$ MHT CET 2023 (09 May Shift 1)

Options:

A. 30

B. $\frac{53}{3}$

C. 16

D. 18

Answer: D

Solution:

Given that $\frac{y^2}{2} \leq x \leq y + 4$

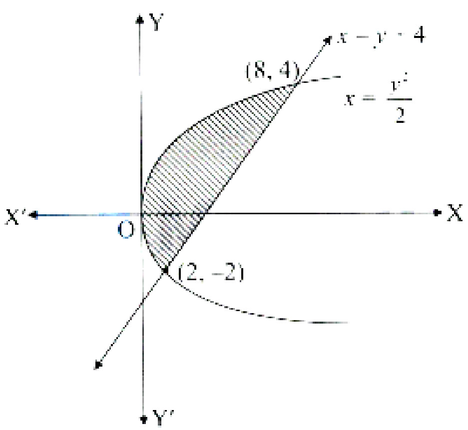
$\therefore \quad x = \frac{y^2}{2}$ and $x = y + 4$

$\frac{y^2}{2} = y + 4^2$

$\therefore \quad y^2 - 2y - 8 = 0$

$\therefore \quad y = 4$ or -2

$\Rightarrow x = 8$ or 2



$\therefore \quad \mathrm{A} = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right) \mathrm{d}y$

$\therefore \quad \mathrm{A} = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$

$\therefore \quad A = \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$

$\therefore \quad \mathrm{A} = 18$

Question46

The area (in sq. units) bounded by the curves $y = \sqrt{x}, 2y - x + 3 = 0$, X-axis and lying in the first quadrant, is MHT CET 2022 (10 Aug Shift 2)

Options:

A. 6

B. $\frac{27}{4}$

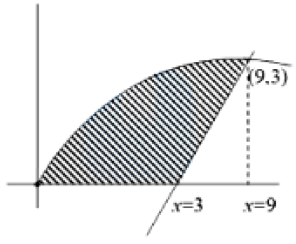
C. 9

D. 18

Answer: C

Solution:

$$\int_0^9 \sqrt{x}\, dx - \frac{1}{2} \times 6 \times 3$$
$$= \frac{2}{3}\left[x^{3/2}\right]_0^9 - 9$$



$= \frac{2}{3} \times 27 - 9 = 9$

Question47

The area bounded by the curve $y^2 = 2x + 1$ and the line $x - y = 1$ is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $\frac{2}{3}$ sq. units

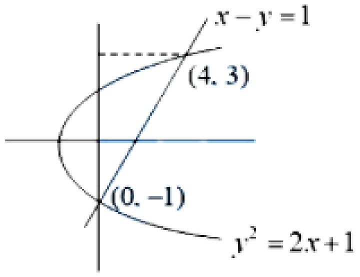
B. $\frac{4}{3}$ sq. units

C. $\frac{8}{3}$ sq. units

D. $\frac{16}{3}$ sq. units

Answer: D

Solution:



Required area $= \int_{-1}^{3}\left\{(1+y)-\left(\frac{y^2-1}{2}\right)\right\} \mathrm{d} y$
$= \int_{-1}^{3}\left(\frac{3}{2}+y-\frac{y^2}{2}\right) \mathrm{d} y$
$= \left[\frac{3}{2} y+\frac{y^2}{2}-\frac{y^3}{6}\right]_{-1}^{3}$
$= \left(\frac{9}{2}+\frac{9}{2}-\frac{27}{6}\right)-\left(\frac{-3}{2}+\frac{1}{2}+\frac{1}{6}\right)=\frac{16}{3}$

Question48

There are (in square units) of the region bounded by the parabola $y=x^2+2$ and the lines $y=x+1, x=0$ and $x=3$ is MHT CET 2022 (08 Aug Shift 2)

Options:

A. $\frac{15}{4}$

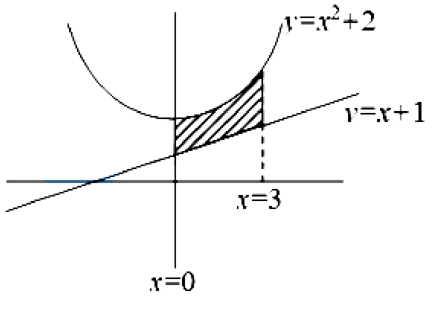
B. $\frac{15}{2}$

C. $\frac{21}{2}$

D. $\frac{17}{4}$

Answer: B

Solution:



$$= \int_0^3\left\{\left(x^2+2\right)-(x+1)\right\} d x$$

$$\text{Required area } = \left[\frac{x^3}{3}+2 x-\frac{x^2}{2}-x\right]_0^3$$

$$= \left(9+6-\frac{9}{2}-3\right)-0=\frac{15}{2}$$

Question49

The area bounded by the curve, $y=-x^2$, x-axis, $x=1$ and $x=4$, is MHT CET 2022 (08 Aug Shift 1)

Options:

A. 21 sq. units

B. 10 sq. units

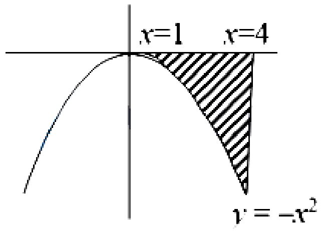
C. 20 sq. units

D. $\frac{21}{2}$ sq. units

Answer: A

Solution:

$-\int_1^4\left(-x^2\right) dx=\left[\frac{x^3}{3}\right]_1^4=\frac{64}{3}-\frac{1}{3}=21$ sq. units



Question50

Area of the region bounded by the curve $y=x^2+2$ and the lines $y=x, x=0$ and $x=3$ is MHT CET 2022 (07 Aug Shift 2)

Options:

A. $\frac{19}{2}$ sq · units

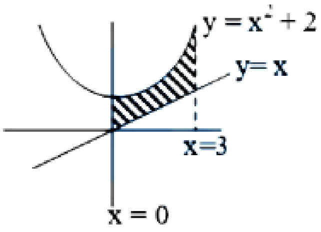
B. $\frac{21}{2}$sq· units

C. 15sq· units

D. $\frac{9}{2}$sq · units

Answer: B

Solution:



Required area $= \int_0^3 (x^2+2-x)\,dx = \left[\frac{x^3}{3}+2x-\frac{x^2}{2}\right]_0^3$

$= \frac{27}{3}+6-\frac{9}{2}=\frac{21}{2}$

Question51

The angle between the curves $y=\sin x$ and $y=\cos x 0

Options:

A. $\tan^{-1}(\sqrt{2})$

B. $\tan^{-1}(3\sqrt{2})$

C. $\tan^{-1}(2\sqrt{2})$

D. $\tan^{-1}(3\sqrt{3})$

Answer: C

Solution:

$y=\sin x$ and $y=\cos x$ intersects at $x=\frac{\pi}{4}$

$\left(\frac{dy}{dx}\right)_{\text{for first curve at } x=\frac{\pi}{4}}=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$\left(\frac{dy}{dx}\right)_{\text{for second curve at } x=\frac{\pi}{4}}=-\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$

Required angle $=\theta=\tan^{-1}\left\{\frac{\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)}{1+\left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)}\right\}=\tan^{-1}\left(\frac{\sqrt{2}}{\frac{1}{2}}\right)$

$=\tan^{-1}(2\sqrt{2})$

Question52

The area of the region bounded by the y-axis, $y = \cos x, y = \sin x$, when $0 \le x \le \frac{\pi}{4}$, is MHT CET 2022 (07 Aug Shift 1)

Options:

A. $(\sqrt{2} - 1)$ **sq. units**

B. $2(\sqrt{2} - 1)$ **sq. units**

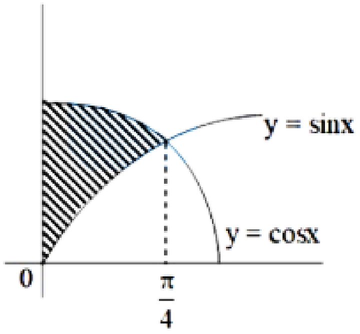
C. $(\sqrt{2} + 1)$ **sq. units**

D. $\sqrt{2}$ **sq. units**

Answer: A

Solution:

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx = [\sin x + \cos x]_0^{\frac{\pi}{4}}$$



$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0 + 1)$$

$$= \sqrt{2} - 1$$

Question53

The area (in sq. units) of the region described by $A = [(x,y)/x^2 + y^2 \leq 1 - x]$ is MHT CET 2022 (06 Aug Shift 2)

Options:

A. $\left(\frac{\pi}{2} - \frac{2}{3}\right)$

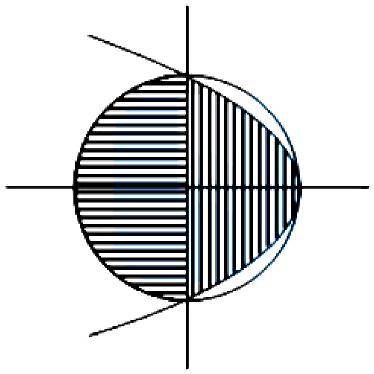
B. $\left(\frac{\pi}{2} + \frac{4}{3}\right)$

C. $\left(\frac{\pi}{2} - \frac{4}{3}\right)$

D. $\left(\frac{\pi}{2} + \frac{2}{3}\right)$

Answer: B

Solution:



$A = \{(x,y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$

$=$ Area of semicircle $+$ Area of the region bounded by parabola and y-axis

$$= \frac{\pi \times 1^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx$$

$$= \frac{\pi}{2} + 2 \times \frac{2}{3}\left[-(1-x)^{3/2}\right]_0^1$$

$$= \frac{\pi}{2} + \frac{4}{3}\left[-0^{3/2} + 1^{3/2}\right]$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

Question54

The area of the region bounded by the curve $y^2 = 9x$ and the line $y = 3x$ is MHT CET 2022 (06 Aug Shift 1)

Options:

A. $\frac{3}{2}$ **sq.units**

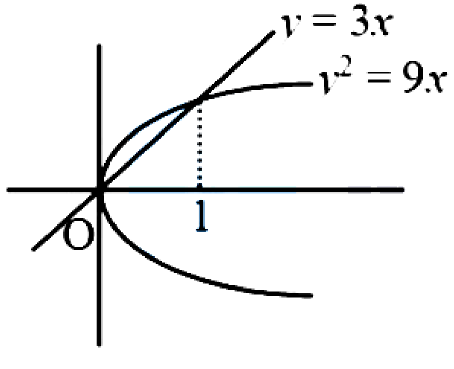
B. 1 sq.units

C. $\frac{1}{2}$ **sq.units**

D. $\frac{1}{4}$ **sq.units**

Answer: C

Solution:



$$\text{Required area } = \int_0^1 (3\sqrt{x} - 3x) dx$$

$$= 3\left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$$
$$= 3\left(\frac{2}{3} - \frac{1}{2}\right)$$
$$= 3 \times \frac{1}{6} = \frac{1}{2}$$

Question55

The area of the region bounded by the line $2y + x = 8, x-$ axis and the lines $x = 2$ and $x = 4$ is MHT CET 2022 (05 Aug Shift 2)

Options:

A. 6 sq. units

B. 5 sq. units

C. 4 sq. units

D. 10 sq. units

Answer: B

Solution:

The required area $= \int_2^4 \frac{8-x}{2} dx$

$$= \left[4x - \frac{x^2}{4}\right]_2^4 = (16 - 4) - (8 - 1) = 5 \text{ sq unit}$$

Question56

The area of the region bounded by the curve $y = 2x - x^2$ and X-axis is MHT CET 2021 (24 Sep Shift 2)

Options:

A. $\frac{2}{3}$ sq. units

B. $\frac{4}{3}$ sq. units

C. $\frac{5}{3}$ sq. units

D. $\frac{8}{3}$ sq. units

Answer: B

Solution:

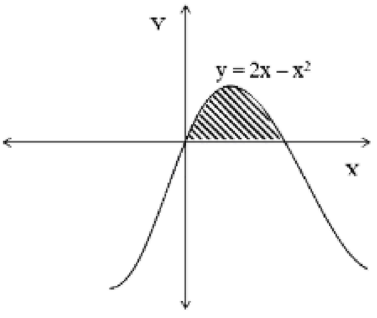
Point of intersection of curve $y = 2x - x^2$ and x axis, is $0 = 2\mathrm{x} - \mathrm{x}^2 \Rightarrow \mathrm{x}(\mathrm{x} - 2) = 0 \Rightarrow \mathrm{x} = 0, 2$

When $\mathrm{x} = 0, \mathrm{y} = 0$ and when $\mathrm{x} = 2, \mathrm{y} = 0$

Refer figure

Required area is shaded

$$A = \int_0^2 (2x - x^2)\, dx$$
$$= 2\left[\frac{x^2}{2}\right]_0^2 - \left[\frac{x^2}{3}\right]_0^2$$
$$= (4) - \left(\frac{8}{3}\right) = \frac{4}{3}$$



Question57

The area of the region include between the parabolas $y^2 = 8x$ and $x^2 = 8y$, is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $\frac{128}{3}$ sq. units

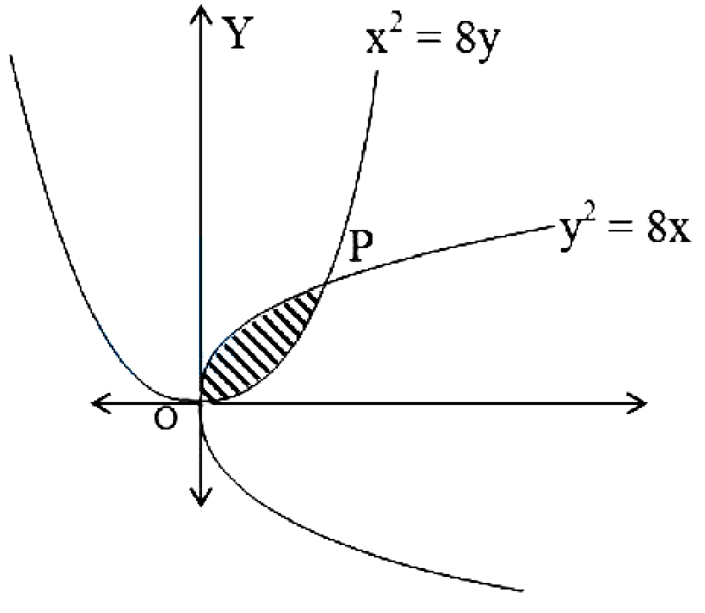
B. $\frac{64}{3}$ sq. units

C. $\frac{32\sqrt{8}}{3}$ sq. units

D. $\frac{16\sqrt{8}}{3}$ sq. units

Answer: B

Solution:



Refer figureRequired area is shaded. Point of intersection of given curves are $y^2 = 8x$ and $x^2 = 8y$ i.e.

$$\left(\frac{x^2}{8}\right)^2 = 8x \Rightarrow x\left(x^3 - 512\right) = 0$$

$$\therefore O \equiv (0,0) \text{ and } P \equiv (8,8)$$

$$A = \int_0^8 (2\sqrt{2})(\sqrt{x})dx - \int_0^8 \frac{x^2}{8}dx$$

$$= \frac{2\sqrt{2}}{\left(\frac{3}{2}\right)} x^{\frac{3}{2}}\Big]_0^8 - \frac{1}{24}\left[x^3\right]_0^8 = \left(\frac{4\sqrt{2}}{3}\right)(8\sqrt{8}) - \frac{1}{24}(512) = \frac{64}{3} \text{ sq. units}$$

Question58

The area bounded by the parabola $\mathrm{y}^2 = 4\mathrm{ax}$ and its latus-rectum $x = a$ is MHT CET 2021 (23 Sep Shift 2)

Options:

A. $\frac{8}{3}a^2$ **sq. units**

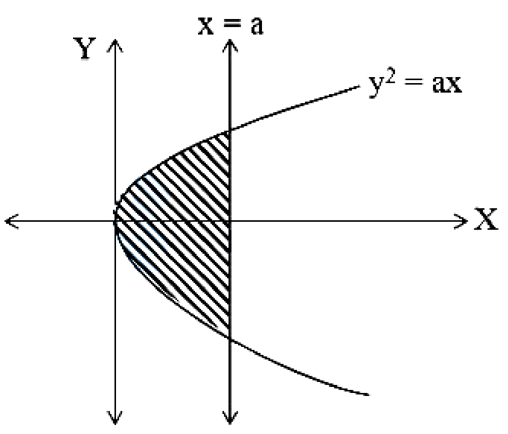
B. $\frac{2}{3}a^2$ **sq. units**

C. $\frac{4}{3}a^2$ **sq. units**

D. $8\mathrm{a}^2$ **aq. units**

Answer: A

Solution:



Required area is shaded.

Point of intersection of $x = a$ and $y^2 = 4ax$, is

$$y^2 = 4a^2 \Rightarrow y = \pm 2a \text{ and } x = a \Rightarrow (a, \pm 2a)$$

$$\therefore \mathrm{A} = 2\int_0^a (2\sqrt{a}\sqrt{x})dx$$

$$= 4\sqrt{a}\int_0^a x^{\frac{1}{2}}dx = 4\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^a = (4\sqrt{a})\left(\frac{2}{3}\right)(a\sqrt{a}) = \frac{8}{3}a^2 \text{ sq. units}$$

Question59

Area bounded by the lines $y = x, x = -1, x = 2$ and the X-axis is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $\frac{1}{2}$ **sq. units**

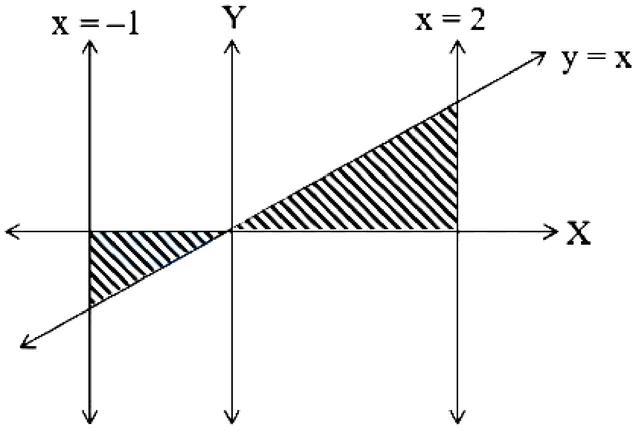
B. $\frac{3}{2}$ **sq. units**

C. $\frac{5}{2}$ **sq. units**

D. $\frac{7}{4}$ **sq. units**

Answer: C

Solution:



Required area shaded

$$A=\int_{-1}^{0} x d x+\int_{0}^{2} x d x$$

$$=\left|\left[\frac{x^2}{2}\right]_{-1}^{0}\right|+\left|\left[\frac{x^2}{2}\right]_{0}^{2}\right|=\frac{1}{2}+2=\frac{5}{2}$$

Question60

The area bounded by the parabola $y^2=x$ and the line $x+y=2$ in the first quadrant is MHT CET 2021 (22 Sep Shift 2)

Options:

A. $\frac{7}{6}$ aq. units

B. $\frac{1}{6}$ sq. units

C. $\frac{2}{3}$ sq. units

D. $\frac{6}{7}$ aq. units

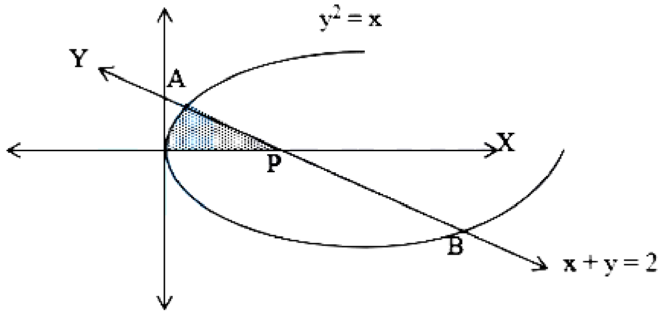
Answer: A

Solution:

The point of intersection of $y^2=x$ and $x+y=2$ is,
$(2-x)^2=x \Rightarrow x^2-5x+4=0 \Rightarrow (x-4)(x-1)=0$

Let $\mathrm{A}=(1,1)$ in first quadrant and $\mathrm{B}=(4,-2)$ in fourth quadrant The line $\mathrm{x}+\mathrm{y}=2$ cuts X axis at $\mathrm{P}(2,0)$

Refer figure



Required area is shaded

$$\therefore A=\int_0^1 \sqrt{x}dx+\int_1^2(2-x)dx$$
$$=\left[\frac{x^{\frac{2}{3}}}{\left(\frac{3}{2}\right)}\right]_0^1+[2x]_1^2-\left[\frac{x^2}{2}\right]_1^2$$
$$=\left[\left(\frac{2}{3}\right)(1)\right]+[2(2-1)]-\left[\left(\frac{4-1}{2}\right)\right]=\frac{2}{3}+2-\frac{3}{2}$$
$$=\frac{7}{6}\text{ sq. units}$$

Question61

The area bounded between the curve $x^2=y$ and the line $y=4x$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $\frac{32}{3}$ sq. units

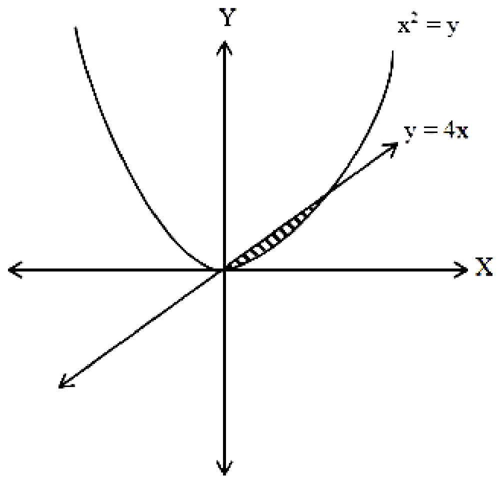
B. $\frac{8}{3}$ sq. units

C. $\frac{1}{3}$ sq. units

D. $\frac{16}{3}$sq. units

Answer: A

Solution:



Required area is shaded.Point of intersection of given curves are $(0,0)$ and $(4,16)$

$$\therefore A=\int_0^4\left(4x-x^2\right)dx$$
$$=4\int_0^4 x dx-\int_0^4 x^2 dx=4\left[\frac{x^2}{2}\right]_0^4-\left[\frac{x^3}{3}\right]_0^4$$
$$=2(16)-\frac{(4)^2}{3}=32-\frac{64}{3}=\frac{32}{3}\text{ sq. units}$$

Question62

The area bounded by the parabola $y^2=x$, the straight line $y=4$ and Y axis is MHT CET 2021 (21 Sep Shift 2)

Options:

A. $2\sqrt{7}$ **sq. unit**

B. $\frac{64}{3}$ **sq. units**

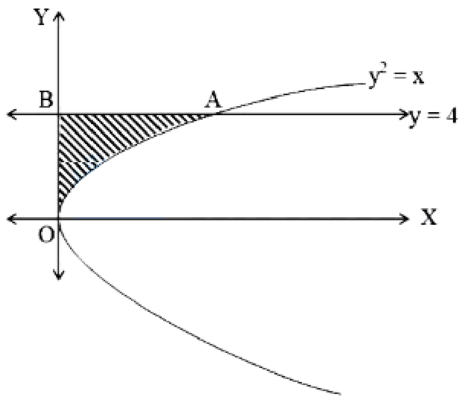
C. $\frac{16}{3}$ **sq. units**

D. $7\sqrt{2}$ **sq. units**

Answer: B

Solution:

Refer figure



Required area is shaded.

Point of intersection of $y^2 = x$ and $y = 4$ is $A \equiv (16, 4)$

$$\therefore A = \int_0^4 y^2 dy$$

$$= \left[\frac{y^3}{3}\right]_0^4 = \frac{64}{3} \text{ sq. units}$$

Question63

The area of the region bounded by the curve $y^2 = 4x$ and the line $y = x$ is MHT CET 2021 (21 Sep Shift 1)

Options:

A. $\frac{8}{3}$ sq. units

B. $\frac{5}{8}$ sq. units

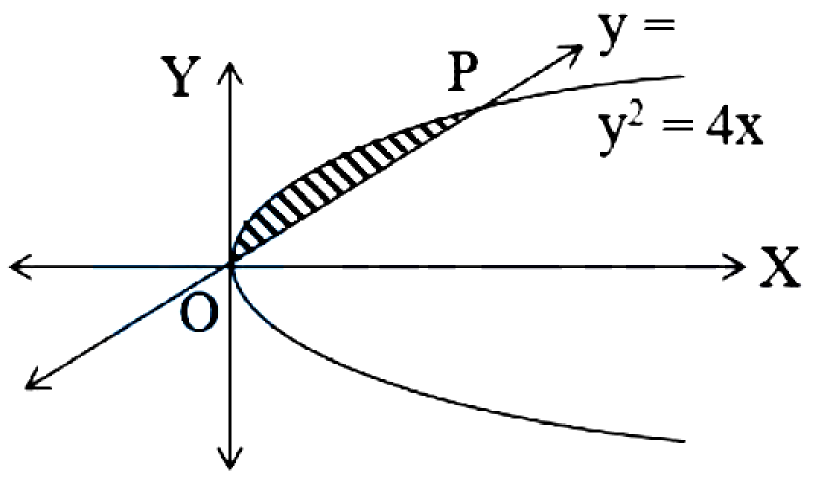
C. $\frac{3}{8}$ sq. units

D. $\frac{3}{5}$ sq. units

Answer: A

Solution:

Refer figure, point of intersection of given curves are $x^2 = 4x \Rightarrow x(x-4) = 0$



$$\therefore O \equiv (0, 0) \text{ and } P \equiv (4, 4)$$

Required area is shaded.

$$\therefore A = \int_0^4 (\sqrt{4x} - x) dx = 2 \int_0^4 x^{\frac{1}{2}} dx - \int_0^4 x dx$$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^4 - \left[\frac{x^2}{2}\right]_0^4 - \left(\frac{4}{3}\right)(4 \times 2) - \frac{16}{2}$$

$$= \frac{32}{3} - \frac{16}{2} = \frac{16}{6} = \frac{8}{3} \text{ sq. units.}$$

Question64

The area bounded by the parabola $y = x^2$ and the line $y = x$ is MHT CET 2021 (20 Sep Shift 2)

Options:

A. $\frac{1}{2}$ sq. units

B. $\frac{1}{3}$ sq. units

C. $\frac{2}{3}$ sq. units

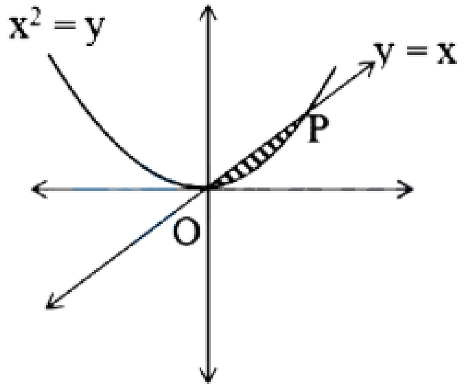
D. $\frac{1}{6}$ sq. units

Answer: D

Solution:

The required area is shaded The point of intersection of the curves are $x^2 = x \Rightarrow x(x-1) = 0$ i.e.

$O(0,0)$ and $P(1,1)$



$$\therefore A = \int_0^1 (x - x^2)\, dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Question65

The area bounded by the parabola $y = x^2$ and the line $y = x$ is MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\frac{1}{2}$ sq. units

B. $\frac{1}{3}$ sq. units

C. $\frac{2}{3}$ sq. units

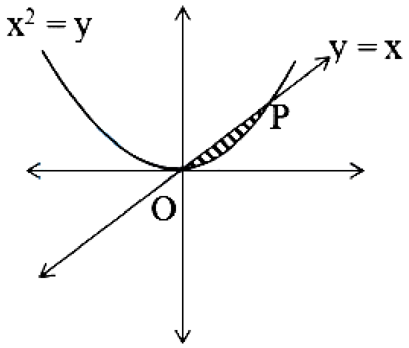
D. $\frac{1}{6}$ sq. units

Answer: D

Solution:

The required area is shaded

The point of intersection of the curves are $x^2 = x \Rightarrow x(x-1) = 0$ i.e. $O(0,0)$ and $P(1,1)$



$\therefore A = \int_0^1 (x - x^2)\, dx$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Question66

The area bonded by the curve $y = \sin^2 x$, x -axis and the lines $x = 0$ and $x = \frac{\pi}{2}$ is MHT CET 2020 (20 Oct Shift 2)

Options:

A. 1 sq. units

B. $\frac{\pi}{8}$ sq. units

C. $\frac{\pi}{4}$ sq. units

D. $\frac{\pi}{2}$ sq. units

Answer: C

Solution:

Required area is shaded.

$$A = \int_0^{\pi/2} \sin^2 x dx$$

$$= \int_0^{\pi/2} \frac{1 - \cos 2x}{2} dx$$
$$= \frac{1}{2}[x]_0^{\frac{\pi}{2}} - \frac{1}{4}[\sin 2x]_0^{\frac{\pi}{2}}$$
$$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Question67

The area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$ in the first quardrant is MHT CET 2020 (20 Oct Shift 1)

Options:

A. $\frac{1}{6}$ **sq. units**

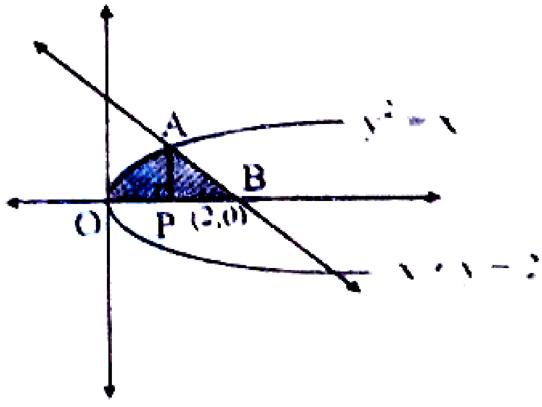
B. $2\frac{7}{6}$sq. **units**

C. $\frac{1}{2}$sq. **units**

D. $\frac{2}{3}$ **sq. units**

Answer: B

Solution:



Point of intersection of $y^2 = x$ and $x + y = 2$ is

$$(2-x)^2 = x \quad \Rightarrow x^2 - 4x - x + 4 = 0$$

$$x^2 - 5x + 4 = 0 \Rightarrow (x-4)(x-1) = 0 \Rightarrow x = 1, 4$$

But since we want area in 1^{st} quadrant only, we take $x = 1 \therefore y^2 = 1 \Rightarrow y = \pm 1 \Rightarrow y = 1$ in 1^{st} quadrant.

$\therefore A \equiv (1,1)$ and $P \equiv (1,0)$

Point of intersection of $x + y = 2$ with X axis is $B = (2,0)$

Hence area required is

$$\begin{aligned} &= \int_0^1 \sqrt{x}dx + \int_1^2 (2-x)dx \\ &= \frac{2}{3}[x\sqrt{x}]_0^1 + 2[x]_1^2 - \frac{1}{2}\left[x^2\right]_1^2 \\ &= \frac{2}{3} + 2 - \left(\frac{1}{2} \times 3\right) = \frac{2}{3} + 2 - \frac{3}{2} = \frac{4+12-9}{6} = \frac{7}{6} \end{aligned}$$

Question68

The area of the triangle formed by the lines joining vertex of the parabola $x^2 = 12y$to the extremities of its latus rectum is MHT CET 2020 (19 Oct Shift 2)

Options:

A. 38 sq. units

B. 18 sq. units

C. 12 sq. units

D. 28 sq. units

Answer: B

Solution:

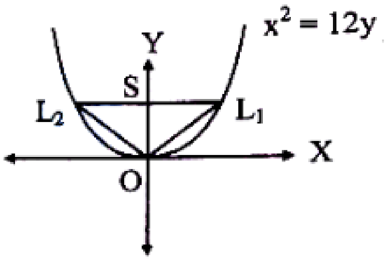
$x^2 = 12y \Rightarrow 4b = 12 \Rightarrow b = 3$

Co-ordinates of latus return are $(\pm 2b, b)$

$\therefore \mathrm{L}_1 \equiv (6, 3)$ and $\mathrm{L}_2 \equiv (-6, 3)$

Coordinates of focus $S \equiv (0, 3)$

$\therefore \mathrm{A}\left(\Delta \mathrm{OL}_1 \mathrm{~L}_2\right) = \frac{1}{2} \times 12 \times 3 = 18$



Question69

The area of the region bounded by the curve $y = \sin x$ between $x = -\pi$ and$x = \frac{3\pi}{2}$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. 2 (unit) ^{2}

B. 5 (unit) ^{2}

C. 3 (unit) ^{2}

D. 1 (unit) ^{2}

Answer: B

Solution:

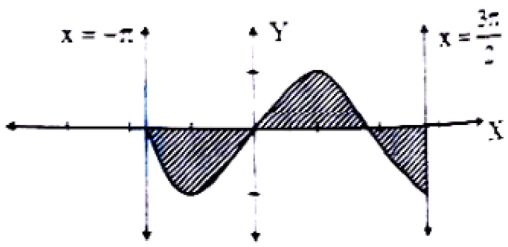
Required area is shaded :

$$A = 2\int_0^{\pi} \sin x dx + \int_{\pi}^{3\pi/2} \sin x dx$$
$$= 2\left[-\cos x|_0^{\pi} + [-\cos x]_{\pi}^{\frac{3\pi}{2}}\right.$$

$$= |2[-\cos\pi + \cos 0]| + \left[-\cos\left(\frac{3\pi}{2}\right) + \cos\pi\right]$$

$$= 2[-(-1) + 1] + [0 + (-1)]$$

$$= 2(2) - (1) = 5(\text{ unit })^2$$



Question70

The area of the region bounded by the parabola $x^2 = 16y, y = 1, y = 4$ and theY-axis lying in the first quartrant is MHT CET 2020 (19 Oct Shift 1)

Options:

A. $\frac{55}{3}$ **sq. units**

B. $\frac{56}{3}$ **sq. units**

C. $\frac{52}{3}$ **sq. units**

D. $\frac{53}{3}$ **sq. units**

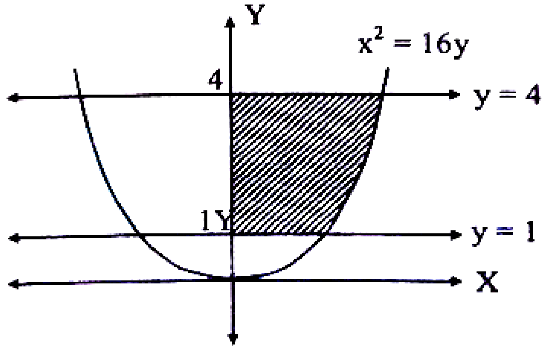
Answer: B

Solution:

The equation of curve is $x^2 = 16y \Rightarrow x = 4\sqrt{y}$ Required area is shaded.

$$A = \int_1^4 4\sqrt{y}dy = 4\int_1^4 y^{\frac{1}{2}}dy = 4\left[\frac{y^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_1^4$$

$$= \frac{8}{3}(4\sqrt{4} - 1) = \frac{8 \times 7}{3} = \frac{56}{3}$$



Question71

The area included between the parabolas $y^2 = 5x$ and $x^2 = 5y$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. $\frac{25}{7}$ **sq. units**

B. $\frac{25}{3}$ **sq. units**

C. $\frac{25}{4}$ **sq. units**

D. 25sq. **units**

Answer: B

Solution:

Prabolas$y^2 = 5x$ and $x^2 = 5y$ intersect at

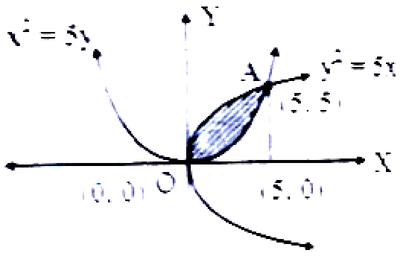
$\frac{x^2}{5} = 5x \Rightarrow x^* = 5^3 x \Rightarrow x\left(x^3 - 125\right) = 0 \Rightarrow x = 0.5 \Rightarrow y = 0, 5$

Let points of intersection be $Q(0, 0)$ and $A(5.5)$. Required area is shaded. Ares included becween the parabolas

$= \int \sqrt{5x} - \frac{x}{5} dx$

$= \frac{\sqrt{5x}}{3} - \frac{x^3}{15} = \frac{2}{3}\Big) (\sqrt{5})(5\sqrt{5}) - \frac{125}{15}$

$= \frac{50}{3} - \frac{125}{15} = \frac{25}{3}$



Question72

The area of the region bounded by the curve $y = 4x^3 - 6x^2 + 4x + 1$ and the lines $x = 1, x = 5$ and x axis is MHT CET 2020 (16 Oct Shift 1)

Options:

A. 428 sq. units

B. 400 sq. units

C. 334 sq. units

D. 378 sq. units

Answer: A

Solution:

Required area is shaded.

$\int_1^5 4x^3-6x^2+4x+1dx$

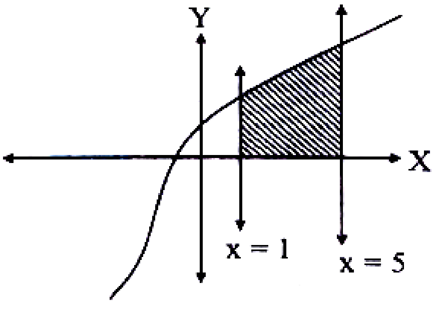
$=\left[\frac{4x^4}{4}-\frac{6x^3}{3}+\frac{4x^2}{2}+x\right]_1^5$

$=\left[x^4-2x^3+2x^2+x\right]_1^5$

$=\left[5^4-2(5)^3+2(5)^2+5\right]-\left[1^4-2(1)^3+2(1)^2+1\right]$

$=[625-250+50+5]-[1-2+2+1]$

$=430-2=428$ sq. units



Question73

The area of the region bounded by the curve y $=x^2+1$, the lines x $=1$, x $=2$ and the X - axis is MHT CET 2020 (15 Oct Shift 2)

Options:

A. $\frac{13}{3}$ **sq. units**

B. $\frac{10}{3}$ **sq. units**

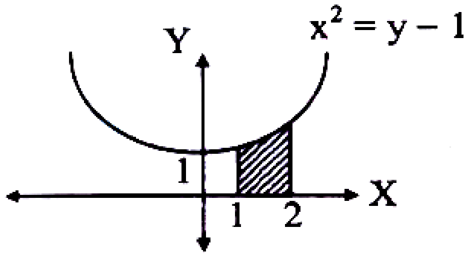
C. $\frac{16}{3}$ **sq. units**

D. $\frac{19}{3}$ **sq. units**

Answer: B

Solution:

$$A=\int_1^2 (x^2+1)\,dx$$
$$=\left[\frac{x^3}{3}+x\right]_1^2=\left(\frac{8}{3}+2\right)-\left(\frac{1}{3}+1\right)=\frac{10}{3}$$



Question74

The area bounded by the parabola $y^2 = 16x$ and its latus - rectum in the first quadrant is MHT CET 2020 (15 Oct Shift 1)

Options:

A. 128 **sq. units**

B. $\frac{64}{3}$ **sq. units**

C. $\frac{128}{3}$ **sq. units**

D. 64 **sq. units**

Answer: B

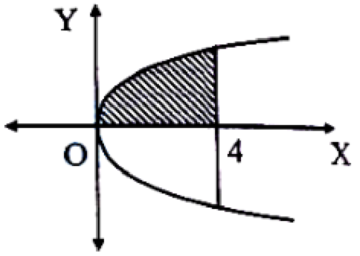
Solution:

We have parabola $y^2=16x \Rightarrow 4a=16 \Rightarrow a=4$.

Hence coordinates of end points of latus rectum are $(4,\pm 8)$

Required area is shaded.

$$\text{Area } = 4\int_0^4 \sqrt{x}\text{dx} = 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$
$$= 4\times\frac{2}{3}\left[4^{\frac{3}{2}}-0\right] = 4\times\frac{2}{3}\times 8 = \frac{64}{3} \text{ sq. units}$$



Question75

The area bounded by the circle $x^2 + y^2 = 16$ and lines x=0 and x=2 is MHT CET 2020 (14 Oct Shift 2)

Options:

A. $\left[4\sqrt{3} + \frac{8\pi}{3}\right]$ **sq. units**

B. $\frac{1}{2}\left[4\sqrt{3} + \frac{8\pi}{3}\right]$ **sq. units**

C. $\left[4\sqrt{3} - \frac{8\pi}{3}\right]$ **sq. units**

D. $\frac{1}{2}\left[4\sqrt{3} - \frac{8\pi}{3}\right]$ **sq. units**

Answer: A

Solution:

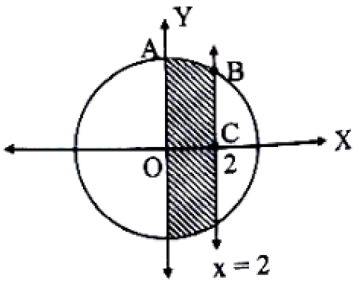
Given equation of circle is $x^2 + y^2 = 16$

$\therefore y^2 = 16 - x^2 \therefore y = \sqrt{16 - x^2}$

Required area is shaded.

$A = 2\ \mathrm{A(OABCO)}$

$$\begin{aligned} \text{Area } &= 2\int_0^2 \sqrt{16 - x^2}\mathrm{dx} \\ &= 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^2 \\ &= 2\left\{\frac{2}{2}\sqrt{12} + 8\sin^{-1}\frac{1}{2} - 0\right\} = 2\left[2\sqrt{3} + 8\left(\frac{\pi}{6}\right)\right] = 4\sqrt{3} + \frac{8\pi}{3} \end{aligned}$$



Question76

The area of the region bounded by the curve $y=\log x, x$ -axis and the lines$x=1, x=e$ is MHT CET 2020 (14 Oct Shift 1)

Options:

A. $\frac{1}{e}$ Sq. Units

B. 1 Sq. Units

C. 4 sq. Units

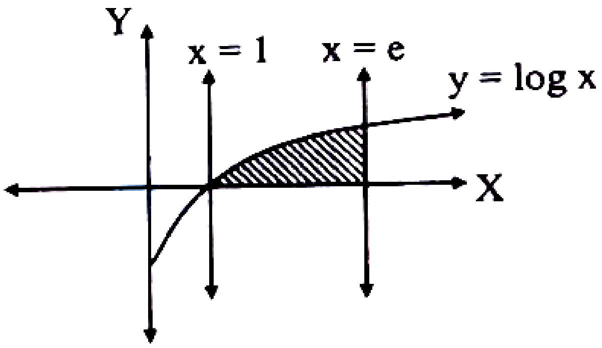
D. $\frac{1}{2}$ Sq. Units

Answer: B

Solution:

Required area is shaded.

$$\begin{aligned} \therefore \mathrm{A} &= \int_1^{\mathrm{e}} \log \mathrm{x} \mathrm{dx}=\int_1^{\mathrm{e}}(\log \mathrm{x})(1) \mathrm{dx} \\ &= [\mathrm{x} \log (\mathrm{x})]_1^{\mathrm{e}}-\int_1^{\mathrm{e}} \frac{1}{\mathrm{x}} \times \mathrm{xdx} \\ &= [\mathrm{e}(\log \mathrm{e})-0]-[\mathrm{x}]_1^{\mathrm{e}}=\mathrm{e}-(\mathrm{e}-1)=1 \end{aligned}$$



Question77

The area of the region bounded by the curve $y=4x-x^2$ and the x -axis is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $\frac{16}{3}$ **sq. units**

B. $\frac{32}{3}$ **sq. units**

C. 32 sq. units

D. 16 sq. units

Answer: B

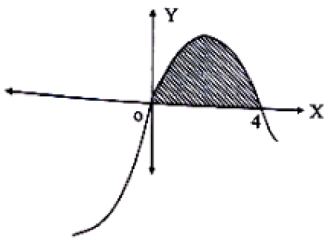
Solution:

We have $y=4x-x^2$

When $y=0$, we get $x(4-x)=0 \Rightarrow x=0,4$

Required area is shaded.

$$\therefore A=\int_0^4\left(4x-x^2\right)dx$$
$$=\left[\frac{4x^2}{2}-\frac{x^3}{3}\right]_0^4=\left[2x^2-\frac{x^3}{3}\right]_0^4$$
$$=\left|2(16-0)-\frac{64-0}{3}\right|=\left|32-\frac{64}{3}\right|$$
$$=\frac{32}{3}$$



Question78

The area bounded by the parabola $x^2 = 4y$ and the lines $y = 2, \quad y = 4$ and Y -axis is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\frac{4}{3}(8 - 2\sqrt{2})$ **sq. units**

B. $\frac{8}{3}(8 - 2\sqrt{2})$ **sq. units**

C. $\frac{8}{3}(8 + 2\sqrt{2})$ **sq. units**

D. $(8 - 2\sqrt{2})$ **sq. units**

Answer: B

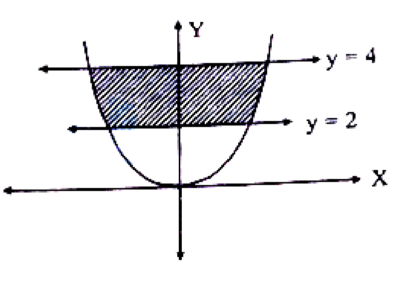
Solution:

$$A = 2\int_2^4 (2\sqrt{y})dy$$

Required area is shaded.

$$= 4\int_2^4 y^{\frac{1}{2}}dy = 4\left[\frac{y^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_2^4 = \frac{8}{3}[4\sqrt{4} - 2\sqrt{2}]$$

$$= \frac{8}{3}(8 - 2\sqrt{2})$$



Question79

The area bounded by the curve $y = x^3$, the X -axis and the lines $x = 1$ and $x = 4$ is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\frac{127}{4}$ sq. units

B. 64 sq. units

C. 27 sq. units

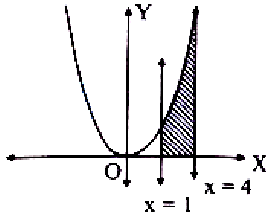
D. $\frac{255}{4}$ sq. units

Answer: D

Solution:

Required area is shaded.

$$\text{Area } = \int_1^4 x^3 dx = \left[\frac{x^4}{4}\right]_1^4$$
$$= \frac{1}{4}\left[4^4 - 1\right] = \frac{1}{4}(256 - 1) = \frac{255}{4} \text{ sq. unit}$$



Question80

The area of the region bounded by the parabola $y^2 = 8x$ and its latus rectum is MHT CET 2020 (12 Oct Shift 1)

Options:

A. $\frac{16}{3}$ sq. units

B. $\frac{8}{3}$ sq. units

C. $\frac{32}{3}$ **sq. units**

D. $\frac{4}{3}$ **sq. units**

Answer: C

Solution:

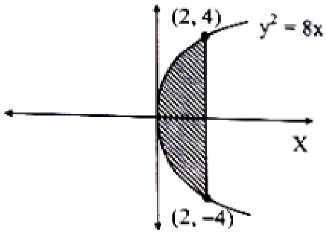
We have parabola $y^2 = 8x \Rightarrow 4a = 8 \Rightarrow a = 2$

Hence coordinates of latus rectum are

$= (a, \pm 2a)$ i.e. $(2, 4)$ and $(2, -4)$

Required area is shaded in figure.

$$\therefore A = 2\int_0^2 \sqrt{8x}\,dx = 4\sqrt{2}\int_0^2 x^{\frac{1}{2}}\,dx$$
$$= 4\sqrt{2}\left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^2 = \frac{8\sqrt{2}}{3}(2\sqrt{2}) = \frac{32}{3}$$



Question81

The area of the region bounded by the curve $y = 2x - x^2$ and the line $y = x$ is ________ square units. MHT CET 2019 (02 May Shift 1)

Options:

A. $\frac{1}{6}$

B. $\frac{1}{2}$

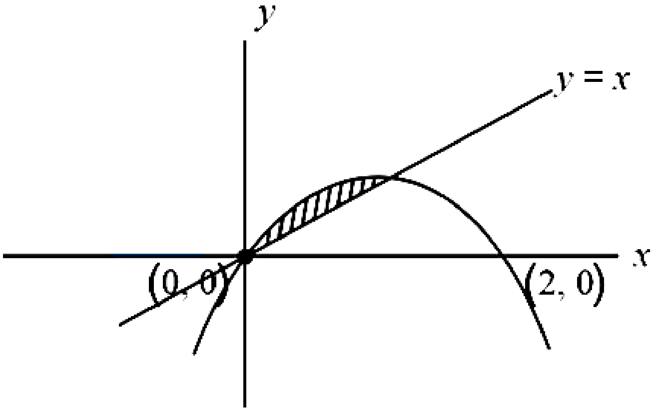
C. $\frac{1}{3}$

D. $\frac{7}{6}$

Answer: A

Solution:

Given curves $y=2x-x^2$ **and** $y=x$



$y=x$
$y=2x-x^2$
Points of intersection of curve
$y=2x-x^2$ **and** $y=x$ **is** $x=0$, **and** $x=1$
Hence, required area $=\int_0^1(y_1-y_2)dx$
$\int_0^1\left[\left(2x-x^2\right)-x\right]dx$
$\left(\frac{x^2}{2}-\frac{x^3}{3}\right)_0^1=\frac{1}{6}$ **sq. unit**

Question82

The maximum value of $z=9x+11y$ **subject to** $3x+2y\leq 12, 2x+3y\leq 12, x\geq 0, y\geq 0$ **is ________ MHT CET 2019 (02 May Shift 1)**

Options:

A. 44

B. 54

C. 36

D. 48

Answer: D

Solution:

Maximize $z = 9x + 11y$ subject to

$$3x + 2y \leq 12, \quad 2x + 3y \leq 12, \quad x, y \geq 0.$$

Check the corner points of the feasible region:

- Axes intercepts:
  $x = 0 \Rightarrow y \leq 4 \rightarrow (0, 4)$, gives $z = 44$.
  $y = 0 \Rightarrow x \leq 4 \rightarrow (4, 0)$, gives $z = 36$.
- Intersection of the two lines:

$$\begin{cases} 3x + 2y = 12 \\ 2x + 3y = 12 \end{cases} \Rightarrow x = y = \frac{12}{5} = 2.4.$$

Then $z = 9(2.4) + 11(2.4) = 20(2.4) = 48$.

The maximum is $\boxed{48}$.

Question83

The minimum value of $z = 10x + 25y$ subject to $0 \leq x \leq 3, 0 \leq y \leq 3, x + y \geq 5$ is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. 80

B. 95

C. 105

D. 30

Answer: A

Solution:

Feasible region: $0 \leq x \leq 3,\ 0 \leq y \leq 3,\ x + y \geq 5 \rightarrow$ the top-right corner of the 3×3 square.

Check extreme points on $x + y = 5$ within the box: $(2, 3)$ and $(3, 2)$, plus the corner $(3, 3)$.

Compute $z = 10x + 25y$:

- $(2, 3)$: $z = 20 + 75 = 95$
- $(3, 2)$: $z = 30 + 50 = 80$
- $(3, 3)$: $z = 30 + 75 = 105$

Minimum is $\boxed{80}$ at $(3, 2)$.

Question84

For L. P. P, maximize $z=4x_1+2x_2$ subject to $3x_1+2x_2 \geq 9, x_1-x_2 \leq 3, x_1 \geq 0, x_2 \geq 0$ has .... MHT CET 2019 (Shift 2)

Options:

A. Infinite number of optimal solutions

B. Unbounded solution

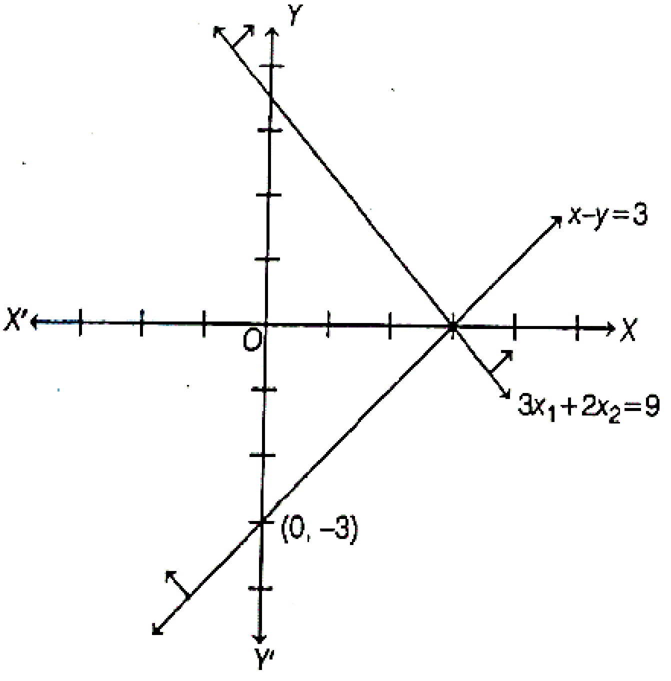
C. No solution

D. One optimal solution

Answer: B

Solution:

We have, maximise $z=4x_1+2x_2$
Subject to constracts, $3x_1+2x_2 \geq 9, x_1-x_2 \leq 3, x_1 \geq 0, x_2 \geq 0$
On taking given constraints as equation, we get the following graphs



Here, we get feasible region is unbounded.

Question85

The area of the region enclosed between pair of line $xy = 0$ and the lines $xy + 5x - 4y - 20 = 0$ is ….. MHT CET 2019 (Shift 2)

Options:

A. 20 square units

B. $\frac{4}{5}$ square units

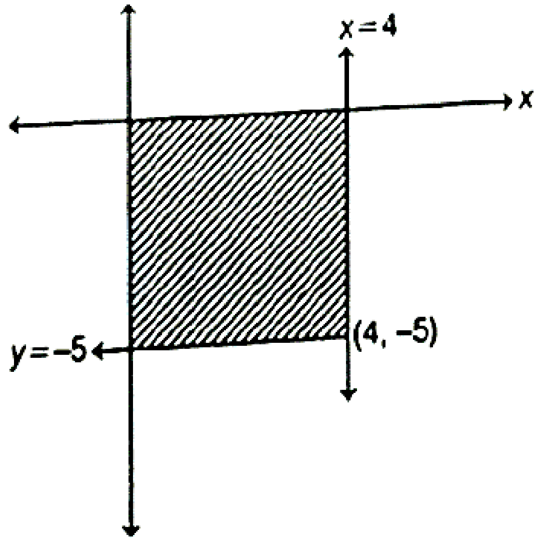
C. 10 square units

D. 6 square units

Answer: A

Solution:

Given pair of lines
$xy + 5x - 4y - 20 = 0$
$\Rightarrow x(y + 5) - 4(y + 5) = 0$
$\Rightarrow (y + 5)(x - 4) = 0$
$\Rightarrow x = 4, y = -5$
and also, $xy = 0$
$\Rightarrow x = 0, y = 0$
Now, draw the graph of the pairs of line



$\therefore$ Required area of bounded region
$= 4 \times 5 = 20 sq. units$

Question86

Area of the region bounded by $y = \cos x, x = 0, x = \pi$ and x-axis is ...sq. units. MHT CET 2019 (Shift 1)

Options:

A. 3

B. 1

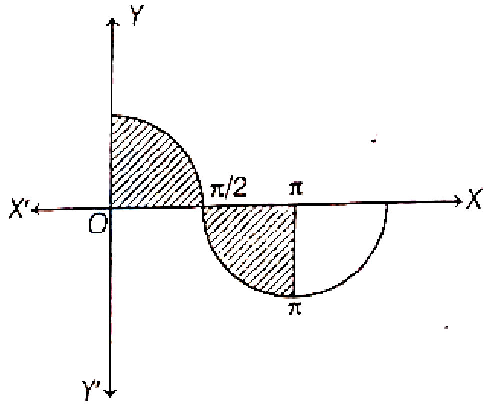
C. 2

D. 4

Answer: C

Solution:

Required area $= 2\int_0^{\frac{\pi}{2}} y dx$
$= 2\int_0^{\frac{\pi}{2}} \cos x dx$
$= 2(\sin x)_0^{\pi/2}$
$= 2(1-0)$
$= 2 \text{sq. units}$



Question87

If $z = ax + by; a, b > 0$ subject to $x \leq 2, y \leq 2, x + y \geq 3, x \geq 0, y \geq 0$ has minimum value at

$(2,1)$ only, then... MHT CET 2019 (Shift 1)

Options:

A. $a > b$

B. $a = b$

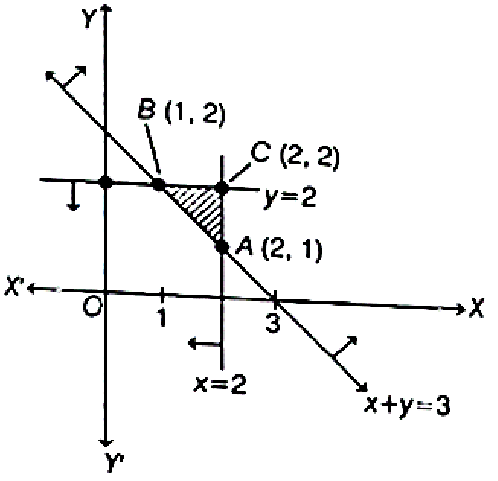
C. $a < b$

D. $a = 1 + b$

Answer: C

Solution:

We have, $z = ax + by, a, b > 0$
Subject to constraints $x \leq 2, y \leq 2, x + y \geq 3x, y \geq 0$
On taking given constraints as equation, we get the following graph



Here, ABCA is the required feasible region whose corner points are $A(2,1), B(1,2)$ **and** $C(2,2)$ **.**
Since, It is given that $z = ax + by; a, b > 0$
has minimum value at $(2,1)$
$\therefore$ **Value of z at** $(2,1) <$ **value of z at** $(1,2)$
$\Rightarrow 2a + b < a + 2b$
$\Rightarrow a < b$

Question88

The maximum value of $Z=5x+4y$, Subject to $y \leq 2x, x \leq 2y, x+y \leq 3, x \geq 0, y \geq 0$ is .. MHT CET 2019 (Shift 1)

Options:

A. 14

B. 12

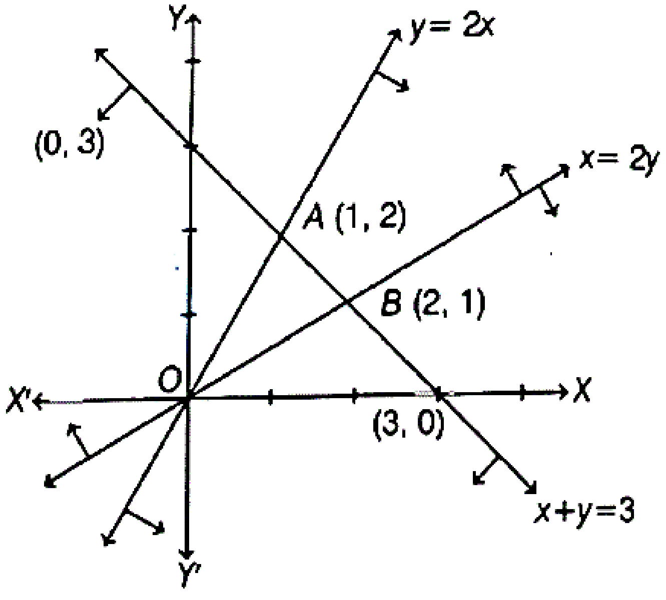
C. 13

D. 18

Answer: A

Solution:

We have, $z=5x+4y$
Subject to contraints $y \leq 2x, x \leq 2y, x+y \leq 3, x \geq 0, y \geq 0$
On taking given constraints as equations, we get the following3 graph.



Intersecting point of line $y=2x$ and $x+y=3$ is $A(1,2)$ and intersecting point of line $y=2y$ and $x+y=3$ is $B(2,1)$
Here OABO is the required feasible region whose corner points are $O(0,0)$, $A(1,2)$ and $B(2,1)$

Corner points	$z=5x+4y$
O,(0,0)	$5 \times 0+4 \times 0=0$
A(1,2)	$5 \times 1+4 \times 2=13$
B(2,1)	$5 \times 2+4 \times 1=14$ (maximum)

Question89

The area of the region bounded by $x^2 = 4y,\ y = 1,\ y = 4$ and the y$-$axis lying in the first quadrant is _______ square units. MHT CET 2018

Options:

A. $\frac{22}{3}$

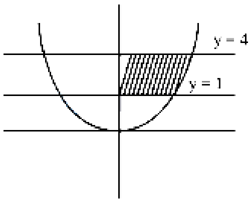
B. $\frac{28}{3}$

C. 30

D. $\frac{21}{4}$

Answer: B

Solution:



Required area A$= \int_1^4 x dy = 2\int_1^4 \sqrt{y} dy$

$A = 2\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4$

$A = \frac{4}{3}(8-1) = \frac{4}{3}(7) = \frac{28}{3}$

Question90

The maximum value of $2x + y$ subject to $3x + 5y \leq 26$ and $5x + 3y \leq 30,\ x \geq 0,\ y \geq 0$ is MHT CET 2018

Options:

A. 12

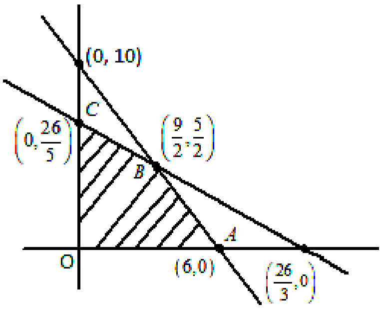
B. 11.5

C. 10

D. 17.33

Answer: A

Solution:



$3x + 5y = 26 \ldots . (i) \times 5$
$5x + 3y = 30 \ldots . (ii) \times 3$
$15x + 25y = 130$
$15x + 9y = 90$
$16y = 40$
$y = \frac{40}{16} = \frac{5}{2}$
$\therefore 3x + \frac{5 \times 5}{2} = 26$
$3x = 26 - \frac{25}{2}$
$x = \frac{9}{2}$
$z = 2x + y$

Now check values of objective function at corner points of the shaded region

$Z_A = 2 \times 6 + 0 = 12$
$Z_B = 2 \times \frac{9}{2} + \frac{5}{2} = 9 + 2.5 = 11.5$
$Z_C = 2 \times 0 + \frac{26}{5} = \frac{26}{5} = 5.2$

Hence $Z_{\max} = 12$ **at** $x = 6$ **and** $y = 0$

Question91

The objective function $Z = 4x_1 + 5x_2$, subject to $2x_1 + x_2 \geq 7,\ 2x_1 + 3x_2 \leq 15, x_2 \leq 3, x_1, x_2 \geq 0$ has

minimum value at the point MHT CET 2017

Options:

A. On $x-$ axis

B. On $y-$ axis

C. At the origin

D. On the line parallel to $x-$ axis

Answer: A

Solution:

Value of $z = 4x_1 + 5x_2$

Convert the given inequalities into equalities to get the corner points $2x_1 + x_2 = 7 \ldots\ldots$ (i)

At $x_1 = 0, x_2 = 7$ and $x_2 = 0, x_1 = 3.5$

So, the corner points of (i) are $(0, 7)$ and $(3.5, 0)$

$2x_1 + 3x_2 = 15 \ldots\ldots$ (ii)

At $x_1 = 0, x_2 = 5$ and $x_2 = 0, x_1 = 7.5$

So, the corner points of (i) are $(0, 5)$ and $(7.5, 0)$

$x_2 = 3$..... (iii)

Plot these corner points on the graph paper and the line given in (iii)

The shaded part shows the feasible region.

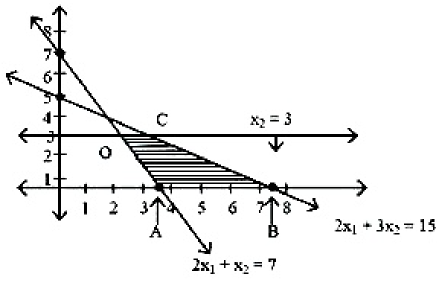
At $x_2 = 3, x_1 = 2$ in (i) and $x_1 = 3$ in (ii)

The corner points of the feasible region are $(3.5, 0), (7.5, 0), (3, 3)$ and $(2, 3)$

Corner points $Z = 4x_2 + 5x_2$

$(3.5, 0)$	14
$(7.5, 0)$	30
$(3, 3)$	27
$(2, 3)$	23

Minimum value of $Z = 14$ and it lies on x-axis.



Question92

The area of the region bounded by the lines $y = 2x + 1, y = 3x + 1$ and $x = 4$ is MHT CET 2017

Options:

A. 16 **sq. unit**

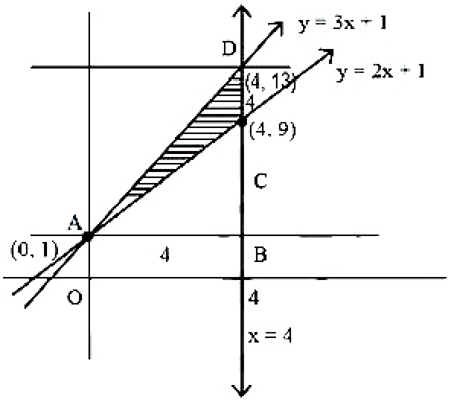
B. $\frac{121}{3}$ **sq.unit**

C. $\frac{121}{6}$ **sq.unit**

D. 8 **sq. unit**

Answer: D

Solution:



A (shaded region)
$= A(\Delta ABD) - A(\Delta ABC) = \frac{1}{2}[4 \times 12 - 4 \times 8] = \frac{1}{2}(48 - 32) = 8$ **sq.units.**
OR

$$A = (\Delta ACD) = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 4 & 9 & 1 \\ 4 & 13 & 1 \end{vmatrix}$$

$$= \frac{1}{2} \times 16 = 8$$

Question93

The area of the region bounded by the curve $y = 2x - x^2$ and $x-$ axis is MHT CET 2016

Options:

A. $\frac{2}{3}$ sq units

B. $\frac{4}{3}$ sq units

C. $\frac{5}{3}$ sq units

D. $\frac{8}{3}$ sq units

Answer: B

Solution:

$$A = \int_0^2 \left(2x - x^2\right) dx$$

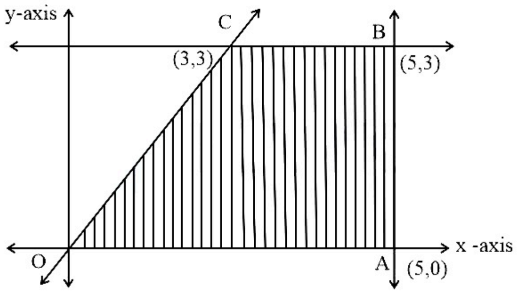
$$\Rightarrow \quad A = \left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$\Rightarrow \quad A = \left[4 - \frac{8}{3}\right]$$

$$\Rightarrow \quad A = \frac{4}{3} \text{ sq units}$$

Question94

The shaded part of given figure indicates the feasible region



Then the constrants are MHT CET 2016

Options:

A. $x,\ y \geq 0,\ x+y \geq 0,\ x \geq 5,\ y \leq 3$

B. $x,\ y \geq 0,\ x-y \geq 0,\ x \leq 5,\ y \leq 3$

C. $x,\ y \geq 0,\ x-y \geq 0,\ x \leq 5,\ y \geq 3$

D. $x,\ y \geq 0,\ x-y \leq 0,\ x \leq 5,\ y \leq 3$

Answer: B

Solution:

Now, as the graph is in first quadrant. So, $x \geq 0, y \geq 0$,

The region lies to the left of the line $\mathbf{x}=5$, So $\mathbf{x} \leq 5$

It lies below the line $y=3$, So $y \leq 3$

Also, the region lies to the right of the line $x=y$. So, $x \geq y$ or $x-y \geq 0$

answer (b)

Question95

The objective function $z=x_1+x_2$, subject to $x_1+x_2 \leq 10, -2x_1+3x_2 \leq 15, x_1 \leq 6, x_1, x_2 \geq 0$ has

maximum value _________ of the feasible region. MHT CET 2016

Options:

A. at only one point

B. at only two points

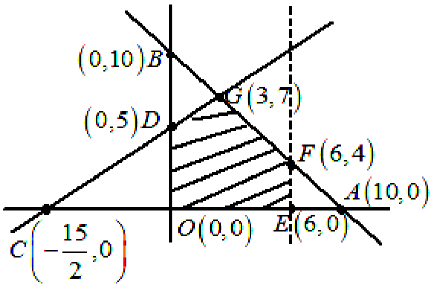
C. at every points of the segment joining two points

D. at every points of the line joining two points

Answer: C

Solution:

The graph of the feasible region for the given L.P.P is



The value of objective function at corner points are

$$z=[x_1+x_2]_O=0$$
$$z=[x_1+x_2]_E=6$$
$$z=[x_1+x_2]_F=10$$
$$z=[x_1+x_2]_G=10$$
$$z=[x_1+x_2]_D=5$$

Since the objective function is maximized at both points F and G, hence it will have a maximum value at every points of the segment joining points F and G

Question96

The area of the region bounded by the curves $x^2 + y^2 = 8$ and$y^2 = 2x$ is MHT CET 2012

Options:

A. $2\pi + \frac{1}{3}$

B. $\pi + \frac{1}{3}$

C. $2\pi + \frac{4}{3}$

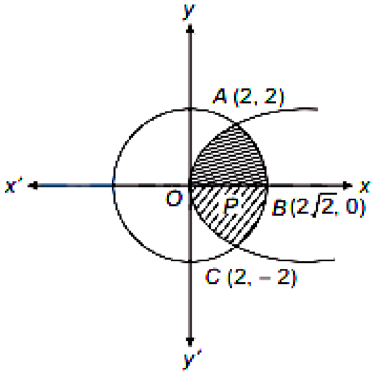
D. $\pi + \frac{4}{3}$

Answer: C

Solution:

Given curves,

$$x^2 + y^2 = 8 \ldots (i)$$
$$and \quad y^2 = 2x \ldots (ii)$$



On solving Eqs. (i) and (ii), we get

$\therefore$

$$x^2 + 2x - 8 = 0$$
$$x^2 + 4x - 2x - 8 = 0$$
$$x(x+4) - 2(x+4) = 0$$
$$(x-2)(x+4) = 0$$
$$x = 2 \text{ and } y = \pm 2$$

∴ Required area

$$= 2[\text{ Area of } OAP + \text{ Area of } PAB]$$
$$= 2\left[\int_0^2 \sqrt{2x}dx + \int_2^{2\sqrt{2}} \sqrt{8-x^2}dx\right]$$
$$= 2\left[\sqrt{2}\left(x^{3/2} \cdot \tfrac{2}{3}\right)_0^2 + \left(\tfrac{x}{2}\sqrt{8-x^2}\right.\right.$$
$$\left.\left.+\tfrac{8}{2}\sin^{-1}\tfrac{x}{2\sqrt{2}}\right)_2^{2\sqrt{2}}\right]$$
$$= 2\left[\tfrac{2\sqrt{2}}{3}\left(2^{3/2}\right) + 4 \times \tfrac{\pi}{2} - 2 - 4 \times \tfrac{\pi}{4}\right]$$

$$= 2\left[\tfrac{2\sqrt{2}}{3} \cdot 2\sqrt{2} + 2\pi - 2 - \pi\right]$$

$$= 2\left[\tfrac{8}{3} - 2 + \pi\right] = 2\left(\tfrac{2}{3} + \pi\right) = 2\pi + \tfrac{4}{3}$$

Question97

The area of the region bounded by the curves, $y^2 = 8x$ and $y = x$ is MHT CET 2012

Options:

A. $\frac{64}{3}$

B. $\frac{32}{3}$

C. $\frac{16}{3}$

D. $\frac{8}{3}$

Answer: B

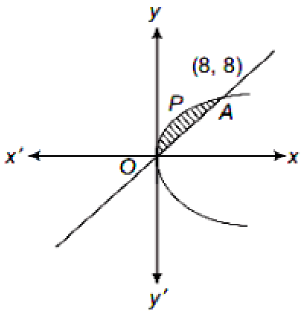
Solution:

Given curves,

$$y^2 = 8x \ldots (i)$$

and

$$y = x \ldots (ii)$$



On solving Eqs. (i) and (ii), we get

$$x^2 - 8x = 0$$
$$x(x-8) = 0$$
$$\Rightarrow \quad x = 0, 8$$

and $\quad y = 0, 8$

$$\therefore \text{ Required area, } (OPA) = \int_0^8 (\sqrt{8x} - x)dx$$
$$= \left[2\sqrt{2} \cdot \frac{2}{3} \cdot x^{3/2} - \frac{x^2}{2}\right]_0^8$$

$$= \frac{4\sqrt{2}}{3} \cdot (8)^{3/2} - \frac{(8)^2}{2}$$

$$= \frac{4\sqrt{2}}{3} \cdot (2)^3 \cdot 2\sqrt{2} - \frac{64}{2}$$

$$= \frac{16}{3} \times 8 - \frac{64}{2} = 64\left(\frac{2}{3} - \frac{1}{2}\right)$$

$$= 64 \times \frac{(4-3)}{6} = 64 \times \frac{1}{6} = \frac{32}{3}$$

Question98

The area bounded by the parabola $y^2 = x$, straight line $y = 4$ and y -axis is square units is MHT CET 2011

Options:

A. $16/3$ **sq. unit**

B. $64/3$ **sq. unit**

C. $7\sqrt{2}$ **sq. unit**

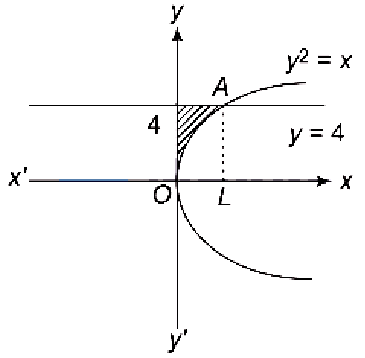
D. None of these

Answer: B

Solution:

Line $y = 4$ meets the parabola $y^2 = x$ at A

$\therefore \quad 16 = x$ and A is $(16, 4)$



$\therefore$ Required area

$$= \int_{y=0}^{4} x dy$$
$$= \int_{y=0}^{4} y^2 dy$$
$$= \left[y^3/3\right]_0^4$$
$$= \frac{64}{3} \text{ sq units}$$

Question99

The volume of the solid formed by rotating the area enclosed between the curve $y^2 = 4x, x = 4$ and $x = 5$ about x -axis is (in cubic units) MHT CET 2010

Options:

A. 18π

B. 36π

C. 9π

D. 24π

Answer: A

Solution:

Volume of the solid $= \int_4^5 \pi y^2 dx$

$= \pi \int_4^5 4x dx$

$= 4\pi \left[\frac{x^2}{2}\right]_4^5$

$= 2x(25 - 16)$

$= 18\pi$ cu unit

Question100

Area bounded between the curve $x^2 = y$ and the line $y = 4x$ is MHT CET 2009

Options:

A. $\frac{32}{3}$sq **unit**

B. $\frac{1}{3}$sq **unit**

C. $\frac{8}{3}$ **sq unit**

D. $\frac{16}{3}$sq **unit**

Answer: A

Solution:

Given curves are $x^2 = y$ and $y = 4x$

Intersection points are $(0, 0)$ and $(4, 16)$

$$\begin{aligned} \therefore \text{ Required area } &= \int_0^4 (4x - x^2)\, dx \\ &= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 \\ &= \left[32 - \frac{64}{3}\right] \\ &= \frac{32}{3} \text{sq unit} \end{aligned}$$

Question101

Area bounded by the lines $y = x, x = -1, x = 2$ and x -axis is MHT CET 2008

Options:

A. $5/2$ **sq unit**

B. $3/2$ **sq unit**

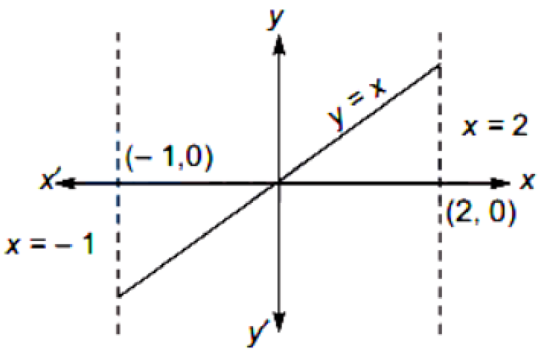
C. $1/2$ **sq unit**

D. None of the above

Answer: A

Solution:

Required area $= \int_{-1}^{2} y dx$



$= \int_{-1}^{0} y dx + \int_0^2 y dx$

$= \left|\int_{-1}^{0} x dx\right| + \int_0^2 x dx$

$= \left| \left.\frac{x^2}{2}\right]_1^0 \right| + \left[\frac{x^2}{2}\right]_0^2 = \frac{5}{2}$ sq unit

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Question102

The volume of solid generated by revolving about the y -axis the figure bounded by the parabola $y=x^2$ and $x=y^2$ is MHT CET 2008

Options:

A. $\frac{21}{5}\pi$

B. $\frac{24}{5}\pi$

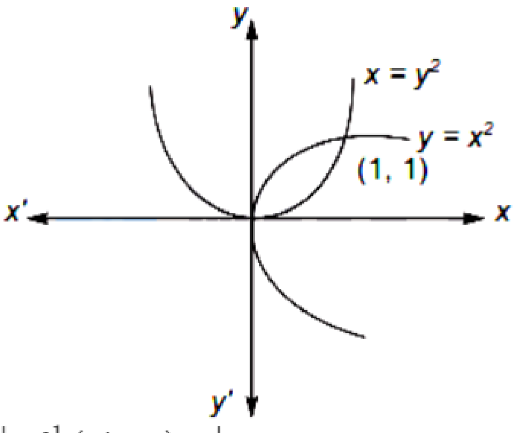
C. $\frac{2}{15}\pi$

D. $\frac{5}{24}\pi$

Answer: C

Solution:

$V=\int_0^1 \pi x^2 dy$



$=\left|\pi \int_0^1\left(y^4-y\right) d y\right|$

$=\left|\pi\left[\frac{y^5}{5}-\frac{y^2}{2}\right]_0^1\right|$

$=\left|\pi\left[\frac{1}{5}-\frac{1}{3}\right]\right|$

$=\left|\frac{-2\pi}{15}\right|=\frac{2\pi}{15}$
